STOCHASTIC MODELING OF RADIATION REGIME IN DISCONTINUOUS VEGETATION CANOPIES

by

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ABSTRACT

The theory of radiative transfer in vegetation canopies is key to modeling of biophysical and biogeochemical processes in the earth system. An outstanding problem is description of radiative transfer in spatially discontinuous vegetation canopies using methods that satisfy the energy conservation principle. This research is based on a stochastic method to describe radiative transfer, which was originally developed in atmospheric physics. Special attention is given to analytical treatment of the effect of spatial discontinuity on the radiation field in discontinuous vegetation canopies. Research indicates that a complete description of the radiation field in discontinuous media is possible using not only average values of radiation over total space, but averages over space occupied by absorbing elements is also required. A new formula for absorbance was obtained for the general case of discontinuous media. Detailed validations of the proposed model was made using available RT models (from simple one-dimensional to complex three-dimensional), Monte Carlo models and field data from shrublands.
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List of Abbreviations

H- total depth (height) of canopy

\( \tilde{\Omega} = \{\Omega_x, \Omega_y, \Omega_z\} \) - unit vector

\( \tilde{\Omega}_0 \) - direction of direct solar radiation

\( \mu(\tilde{\Omega}) \) - cosine of polar angle of \( \tilde{\Omega} \)

\( \delta(\tilde{\Omega} - \tilde{\Omega}_0) \) - Dirac delta function

\( \tilde{r} = \{x, y, z\} \) - coordinate triplet

\( \chi(\tilde{r}) \) - indicator function

\( I(\tilde{r}, \tilde{\Omega}) \) - radiance (intensity) at spatial point \( \tilde{r} \) and in direction \( \tilde{\Omega} \)

\( \bar{I}(z, \tilde{\Omega}) \) - mean radiance, averaged over the horizontal plane at depth \( z \)

\( U(z, \tilde{\Omega}) \) - mean radiance, averaged over the vegetated portion of a horizontal plane \( z \)

\( S_R(x_0, y_0) \) - cylinder of height \( H \), with vertical axis located at point \((x_0, y_0)\) and radius \( R \)

\( T_z \) - area of horizontal plane \( z \), \( z \in [0; H] \) covered by vegetation

\( T_z(x_0, y_0) \) - manifold \( T_z \), shifted by vector \( \{x_0, y_0\} \)

\( S_R \cap T_z \) - common area of manifold \( S_R \) , \( T_z \)

\( \text{Mes}(S) \) - measure of area \( S \)

\( p(z) \) - horizontal density of vegetation (HDV) at level \( z \)

\( u_r(\tilde{r}) \) - foliage area volume density (FAVD), \([ \text{m}^2 / \text{m}^3 \])

\( \lambda \) - wavelength
$r_D(\lambda)$ - spectral hemispherical reflectance of the leaf

$t_D(\lambda)$ - spectral hemispherical transmittance of the leaf

$\omega(\lambda)$ - single-scattering leaf albedo

$\rho_{\text{soil}}(\lambda)$ - soil hemispherical reflectance

$G(\tilde{r}, \tilde{\Omega})$ - mean projection of leaf normals in the direction $\tilde{\Omega}$

$\frac{1}{\pi} \Gamma(\tilde{r}, \tilde{\Omega}' \rightarrow \tilde{\Omega})$ - area scattering phase function

$\bar{\sigma}_s(\tilde{\Omega}' \rightarrow \tilde{\Omega})$ - differential scattering cross-section

$\bar{\sigma}(\tilde{\Omega})$ - extinction coefficient

HDRF- the Hemispherical-Directional Reflectance Factor

BHR- the Bi-Hemispherical Reflectance
1 Introduction

An accurate description of photon transport in vegetation canopies is of interest in many branches of contemporary science: optical remote sensing of vegetated land surfaces, land surface climatology, plant physiology, etc. The development of radiative transfer (RT) theory in vegetation canopies shows a gradual evolution from description of simple homogeneous media to complex discrete media. When the height of canopy is small and the vegetation is evenly distributed on the ground (as it is in the case of crops and grasses), the turbid medium approach of a vegetated canopy is valid and the standard 1D RT equation is used (Ross, 1975). In this case, the canopy is treated as a homogeneous gas with nondimensional planar scattering centers, which are not spatially correlated with each other. But typically the more complex case occurs in nature, when individual vegetation units can be distinguished (individual trees in a forest, for example) and the effect of clustering of vegetation elements become important. The effect of clustering of vegetation, or the phenomenon that positions of vegetation elements tend to be correlated, exists, simply because leaves arise on stems, branches and twigs. An extreme example of lateral heterogeneity is shrubland, which is characterized by low (0.2) to intermediate (0.6) vegetation ground cover (Myneni et al., 1997). The structure of vegetation canopy affects the signature of the radiation field reflected from vegetation canopy (as measured by satellite sensors, for example), and retrieval of biophysical variables from remote observations requires a precise understanding of the signal generating mechanisms. The turbid medium approximation results in poor simulations in cases where horizontal heterogeneity is pronounced, and more precise modeling is required.
The notion of gaps (or voids) between canopy clusters must be introduced along with precise description of topology of the boundary of vegetation in order to describe the signature of radiation field in discontinuous canopy. Nilson (1991), later Li and Strahler (1992), introduced a geometrical-optical approach to calculate the reflected radiance from such vegetation boundaries. They use the notion of mutual shadowing (vegetation unit casting shadows on such units) and Bidirectional Gap Probability (probability of sensing radiation reflected by vegetation along the direction $\Omega$, if it was illuminated by solar radiation along $\tilde{\Omega}$), to describe the boundary of vegetation. This approach allows an explanation for the hot-spot effect (the peak in reflected radiance distribution along the retro-illumination direction, due to the absence of shadows in this direction). The approach is valid in the visible part of solar spectrum, where one can restrict the study of radiation interaction to that scattered once from the boundary only. But in the NIR region, leaf absorption is weak and scattering dominates, and the approach of Nilson/Li and Strahler is not accurate. The problem lies in an accurate description of multiple scattering, and propagation of radiation into deeper parts of the canopy. Currently only the Monte Carlo method (Marshak, Ross, 1991) works well at all wavelengths, but this method has a several disadvantages: these include computational expense, difficulty to adopt to user specific needs, and lack of analytical analysis.

Another analytical approach describing leaf clumping in vegetation canopies is the statistical approach. Of importance is the problem of deriving analytical expressions or equations for moments which characterize the stochastic radiative field in a vegetation canopy. The most critical is the expression for the first moment of the radiation field, the
mean intensity. The problem of investigating the stochastic equations for the mean field has been a highly active research field in recent years (Pomraning, 1995). The first significant attempt to apply statistical approach to describe vegetation canopy was made by Menzhulin and Anisimov (1991). A more manageable closed system of statistical equations for the mean intensity was derived in applications to a medium of broken clouds, initially by Vainikko (1973a, 1973b) and later by Titov (1990), and Zuev and Titov (1996). This approach can be applied to vegetation canopies but with some modifications.

In this thesis, an exact stochastic radiative transfer equation for the mean intensity in a discontinuous vegetation canopy is derived. This equation is based on the work of Vainikko (1973a) for broken clouds together with classical parameters of a vegetation canopy originally introduced by Ross (1975). We obtained a system of integral equations, which were solved numerically. The simulated radiation regime in a discontinuous canopy was validated in several ways, including comparison with field data from Jornada PROVE (Privette et al., 1999).

This thesis is organized as follows. In Section 2 we review the basic concepts of radiative transfer in vegetation media and introduce the classical stochastic 3-dimensional radiative transfer equation with the corresponding boundary conditions. The derivation of the transfer equation for the mean field using a statistical approach is described in Section 3. Issues resulting from the effect of discontinuity in vegetated media and analytical description of this discontinuity (in particular, a new formula for absorptance) are discussed in Section 4. In section 5, a numerical method for solving the transfer equation
for the mean field is outlined and issues related to speed of convergence are presented. Section 6 presents detailed description of important outputs of the model and comparisons with output from similar RTE models, radiation field in Maize canopy simulated using Monte Carlo method and field data from Jornada PROVE. In addition, a numerical study of the effect of discontinuity on the radiation field in a vegetation canopy presented.

2 Classical 3-dimensional Radiative Transfer in Vegetation Canopies

Consider a canopy of depth $H$ in a coordinate system with vertical axis $z$ directed downward (shown on Fig.1). We describe canopy structure with the indicator function

$$
\chi(\vec{r}) = \begin{cases} 
1, & \text{if } \vec{r} \in \text{vegetation}, \\
0, & \text{otherwise}, 
\end{cases}
$$

(1)

where $\vec{r}$ is the coordinate triplet $[\vec{r} = (x, y, z)]$ with its origin at the top of the canopy. The indicator function is treated as a random variable. Its distribution function, in the general case, depends on both macroscale (e.g., random dimensions of the trees and their spatial distribution) and microscale (e.g., structural organization of an individual tree) properties of the vegetation canopy and includes all three of its components, absolutely continuous, discrete, and singular (Knyazikhin et al., 1998c). It is supposed that photons interact with phytoelements only; that is, we ignore photon interactions with the optically active elements of the atmosphere inside the layer $z \in [0, H]$. 

In order to approximate the canopy structure, a fine spatial mesh is introduced by dividing the layer $[0, H]$ into non-overlapping fine cells $e(r)$ of size $\Delta x = \Delta y = \Delta z = \varepsilon$. Each realization $\chi(r)$ of canopy structure is replaced by its mean over fine cell $e(r)$, the foliage area volume density (FAVD), as

$$u_t(\bar{r}) = \frac{1}{\text{Mes}(e(\bar{r}))} \int \chi(\bar{r}) d\bar{r} = \frac{1}{V} \sum_{j=1}^{N} S_j;$$

(2)

here $\text{Mes}(\ldots)$ means measure of cell $e(\bar{r})$ (in most cases is simply volume), $V$ is a volume of $e(\bar{r})$ and $S_j$ is one-sided leaf area. This integration (‘smoothing’) technique provides the convergence process (Knyazikhin et al., 1998c) $u_t(r) \rightarrow \chi(r)$ when $\varepsilon \rightarrow 0$, and so, Eq. (2) can be taken as an approximation of the structure of the vegetation canopy. The accuracy of this approximation depends on size $\varepsilon$ of the fine cell $e(\bar{r})$. To our knowledge, all existing canopy radiation models are based on approximation of Eq. (2) by piece-wise continuous functions, e.g., describing both the spatial distribution of various geometrical objects like cones, ellipsoids, etc., and the variation of leaf area within a geometrical figure (Ross, 1975; Li and Strahler, 1992, Li at al., 1995; Nilson, 1977). Also, we assume that the density of phytoelements in foliated cells is constant, that is

$$u_t(\bar{r}) = d_t \chi(\bar{r}),$$

(3)

where $d_t$ is the one-sided leaf area per unit volume (in $m^2/m^3$).

The vertical heterogeneity of vegetation canopy is described by variation of horizontal density of vegetation with height, referred to later as HDV (in other words, the probability of finding foliage elements at depth $z$) and defined as
\[ p(z) = \frac{1}{S} \int_{x,y} \chi(\vec{r}) \, dx \, dy; \]  

(4)

Here \( S \) means sufficiently large area over the horizontal plane \( z \). In terms of these notations, the leaf area index (LAI) can be expressed as

\[
\text{LAI} = \frac{1}{S} \int_{V} u_{L}(\vec{r}) \, dV = \frac{d_{L}}{S} \int_{V} \chi(\vec{r}) \, dV = d_{L} \int_{0}^{H} \frac{1}{S} \int_{x,y} \chi(\vec{r}) \, dx \, dy = d_{L} \int_{0}^{H} p(\vec{z}) \, d\vec{z}.
\]  

(5)

To describe interaction of canopy elements (leaves) with radiation, we use (Ross, 1975) the mean projection of leaf normals in direction \( \vec{\Omega} \),

\[
G(\vec{r}, \vec{\Omega}) = \frac{1}{2\pi} \int_{2\pi} g_{L}(\vec{r}, \vec{\Omega}_{L}) \left| \vec{\Omega} \cdot \vec{\Omega}_{L} \right| \, d\vec{\Omega}_{L},
\]  

(6)

and the area-scattering phase function

\[
\frac{1}{\pi} \Gamma(\vec{r}, \vec{\Omega}' \rightarrow \vec{\Omega}) = \frac{1}{2\pi} \int_{2\pi} g_{L}(\vec{r}, \vec{\Omega}_{L}) \left| \vec{\Omega}' \cdot \vec{\Omega}_{L} \right| \gamma_{L,\lambda}(\vec{r}, \vec{\Omega}_{L}, \vec{\Omega}' \rightarrow \vec{\Omega}) \, d\vec{\Omega}_{L}.
\]  

(7)

Here \( g_{L}(r, \Omega_{L}) \) is the probability density of leaf normal orientation over the upper hemisphere and

\[
\frac{1}{2\pi} \int_{2\pi} g_{L}(\vec{r}, \vec{\Omega}_{L}) \, d\vec{\Omega}_{L} = 1.
\]

Optical properties of canopy elements are described by the leaf-scattering phase function, \( \gamma_{L,\lambda} \), defined as (Shults and Myneni, 1998)

\[
\gamma_{L,\lambda}(\vec{r}, \vec{\Omega}_{L}, \vec{\Omega} \rightarrow \vec{\Omega}') = \begin{cases}
\frac{1}{\pi} r_{L}(\lambda) \cdot \left| \vec{\Omega} \cdot \vec{\Omega}_{L} \right| \left( \vec{\Omega} \cdot \vec{\Omega}_{L} \right) (\vec{\Omega}' \cdot \vec{\Omega}_{L}) & < 0, \\
\frac{1}{\pi} r_{L}(\lambda) \cdot \left| \vec{\Omega} \cdot \vec{\Omega}_{L} \right| \left( \vec{\Omega} \cdot \vec{\Omega}_{L} \right) (\vec{\Omega}' \cdot \vec{\Omega}_{L}) & > 0.
\end{cases}
\]
Here $r_d(\lambda)$ and $t_d(\lambda)$ are the spectral hemispherical reflectance and transmittance, respectively, of the leaf element. The leaf scattering phase function integrated over all exit photon directions yields the single-scattering leaf albedo (per unit leaf area), $\omega(\lambda)$, i.e.,

$$\int_{4\pi} \gamma_{L,\lambda}(\vec{r}, \vec{\Omega}, \vec{\Omega} \rightarrow \vec{\Omega}') \ d\vec{\Omega}' = \omega(\lambda).$$

With this background information, we can compactly represent the extinction coefficient $\sigma(\vec{\Omega})$ and the differential scattering coefficient $\sigma_s(\vec{\Omega}' \rightarrow \vec{\Omega})$ as (Ross, 1975)

$$u_\perp(\vec{r})G(\vec{r}, \vec{\Omega}) = d_\perp\chi(\vec{r})G(\vec{r}, \vec{\Omega}) = \sigma(\vec{\Omega})\chi(\vec{r}) = \sigma(\vec{r}, \vec{\Omega}), \quad (8)$$

and,

$$\frac{u_\perp(\vec{r})}{\pi} \Gamma(\vec{r}, \vec{\Omega}' \rightarrow \vec{\Omega}) = \frac{d_\perp\chi(\vec{r})}{\pi} \Gamma(\vec{r}, \vec{\Omega}' \rightarrow \vec{\Omega}) = \sigma_s(\vec{\Omega}' \rightarrow \vec{\Omega})\chi(\vec{r}) = \sigma_s(\vec{r}, \vec{\Omega}' \rightarrow \vec{\Omega}). \quad (9)$$

The radiation regime in such a canopy is described by the transport equation (Knyazikhin et al., 1998b)

$$\vec{\Omega} \cdot \nabla I(\vec{r}, \vec{\Omega}) + \chi(\vec{r})\sigma(\vec{\Omega})I(\vec{r}, \vec{\Omega}) = \chi(\vec{r})\int_{4\pi} \sigma_s(\vec{\Omega}' \rightarrow \vec{\Omega})I(\vec{r}, \vec{\Omega}')d\vec{\Omega}'. \quad (10)$$

This equation will be referred later as the classical stochastic radiative transfer equation. It differs from non-stochastic [introduced for turbid media by Ross (1975)] by the indicator function $\chi(\vec{r})$, which modifies the second and third term of Eq. (10).

An important feature of the radiation regime in vegetation canopies is the hot-spot effect, which is the peak in reflected radiance distribution along the retro-illumination direction. The standard theory describes the hot-spot by modifying the extinction coefficient $\sigma(\Omega)$, namely (Marshak, 1989),
\[
\sigma(\vec{\Omega}, \vec{\Omega}_o) = \sigma(\vec{\Omega}) \cdot h(\vec{\Omega}, \vec{\Omega}_o),
\]

where \( h(\vec{\Omega}, \vec{\Omega}_o) \) is

\[
h(\vec{\Omega}, \vec{\Omega}_o) = \begin{cases} 
1 - \frac{G(\vec{\Omega}_o) \mu(\vec{\Omega})}{G(\vec{\Omega}) \mu(\vec{\Omega}_o)} \cdot \exp\left\{ \Delta(\vec{\Omega}, \vec{\Omega}_o) \cdot k \right\}, & \text{if } (\vec{\Omega} \cdot \vec{\Omega}_o) < 0, \\
1, & \text{if } (\vec{\Omega} \cdot \vec{\Omega}_o) > 0,
\end{cases}
\]

(11)

and

\[
\Delta(\vec{\Omega}, \vec{\Omega}_o) = \sqrt{\frac{1}{\mu^2(\vec{\Omega}_o)} + \frac{1}{\mu^2(\vec{\Omega})} + \frac{2(\vec{\Omega}_o \cdot \vec{\Omega})}{\mu(\vec{\Omega}) \mu(\vec{\Omega}_o)}}.
\]

(12)

In the above, \( \mu(\vec{\Omega}) \) denote cosine of polar angle \( \vec{\Omega} \), \( k \) is an empirical parameter, related to the ratio of vegetation height to characteristic leaf dimension. Its value was estimated to be between 1 and 8 based on several sets of experimental data (Stewart, 1990).

In order to specify a unique solution of Eq. (10), it is necessary specify radiance penetrating into the canopy through upper (\( z=0 \)) and lower (\( z=H \)) boundaries. The canopy is illuminated from above by a direct monodirectional solar component in direction \( \vec{\Omega}_o \), \( [\mu_0(\vec{\Omega}_o) < 0] \), namely \( C_x \delta(\vec{\Omega} - \vec{\Omega}_o) \), and diffuse radiation from the sky, \( \tilde{d}(\vec{\Omega}, \vec{\Omega}_o) \). At the ground interface, the corresponding boundary condition is radiation reflected by the ground below the vegetation,

\[
\begin{cases} 
I(0, \vec{\Omega}) = C_x \delta(\vec{\Omega} - \vec{\Omega}_o) + \tilde{d}(\vec{\Omega}, \vec{\Omega}_o), & \mu(\vec{\Omega}) < 0, \\
I(H, \vec{\Omega}) = I_H(\vec{\Omega}), & \mu(\vec{\Omega}) > 0.
\end{cases}
\]

(13)

The canopy bottom is assumed to be a horizontally homogeneous Lambertian surface. In this case the function \( I_H(\vec{\Omega}) \) can be expressed as
\[ I_h(\tilde{\Omega}) = \frac{\rho_{\text{soil}}(\lambda)}{\pi} \int I(H, \tilde{\Omega}) \mu(\tilde{\Omega}) |d\tilde{\Omega}|, \]

where \( \rho_{\text{soil}}(\lambda) \) is the soil hemispherical reflectance.

The incoming radiation can be parameterized in terms of two scalar values:

- \( F_x(\tilde{\Omega}_o) \), total flux defined as

\[
F_x(\tilde{\Omega}_o) = \int I(0, \tilde{\Omega}) |\mu(\tilde{\Omega})| d\tilde{\Omega} = \int \left\{ C_\lambda \delta (\Omega - \Omega_o) + \tilde{d}(\Omega, \Omega_o) \right\} |\mu(\tilde{\Omega})| d\tilde{\Omega} = C_\lambda |\mu(\tilde{\Omega}_o)| + \int \tilde{d}(\Omega, \tilde{\Omega}_o) |\mu(\tilde{\Omega})| d\tilde{\Omega}, \tag{14}
\]

and \( f_{\text{dir}}(\lambda) \), the ratio of direct radiation incident on the top of plant canopy to the total incident irradiance

\[
f_{\text{dir}}(\lambda) = \frac{C_\lambda |\mu(\tilde{\Omega}_o)|}{F_x(\tilde{\Omega}_o)} \in [0;1]. \tag{15}
\]

Equation (14) and Eq. (15) explain the following formula for incoming solar radiation

\[
C_\lambda \delta (\tilde{\Omega} - \tilde{\Omega}_o) + \tilde{d}(\tilde{\Omega}, \tilde{\Omega}_o) = F_x(\tilde{\Omega}_o) \left\{ \frac{f_{\text{dir}}(\lambda)}{|\mu(\tilde{\Omega}_o)|} \delta (\tilde{\Omega} - \tilde{\Omega}_o) + \left[ 1 - \frac{f_{\text{dir}}(\lambda)}{|\mu(\tilde{\Omega}_o)|} \right] I(\tilde{\Omega}, \tilde{\Omega}_o) \right\}. \tag{16}
\]

The general boundary value problem [Eq. (10) and Eq. (13)] can be split into two simpler sub-problems- (1) Black Soil (BS) problem. In this case all energy input is from solar radiation (direct and diffuse) and lower boundary \((z=H)\) is assumed to 100\% absorbing. The problem is defined by Eq. (10) and the boundary condition,
The solution of BS problem is referred to later as \( I_{\lambda}^{BS}(r, \tilde{\Omega}) \); \( I_{\lambda}^{BS}(r, \tilde{\Omega}) \) depends on sun-view geometry, canopy architecture, and spectral properties of the leaves. (2) Soil (S) problem. In this case there is no input of energy from above, but the source intensity \( I(H, \tilde{\Omega}) \) is located at the bottom of the canopy. In the case of Lambertian surface \( I(H, \tilde{\Omega}) = \frac{1}{\pi} \). The S-problem is defined by Eq. (10) and the boundary condition,

\[
\begin{align*}
I(0, \tilde{\Omega}) &= 0, \quad \mu(\tilde{\Omega}) < 0, \\
I(H, \tilde{\Omega}) &= \frac{1}{\pi}, \quad \mu(\tilde{\Omega}) > 0.
\end{align*}
\]

Similarly, the solution of S problem is referred to later as \( I_{\lambda}^{S}(r, \tilde{\Omega}) \); \( I_{\lambda}^{S}(r, \tilde{\Omega}) \) depends on spectral properties of leaves and canopy structure only.

After the solutions of ‘BS’ and ‘S’ problems are obtained, one can construct the solution for the corresponding general case using the following approximations (Knyazikhin et al., 1998b):

for intensity,

\[
I_{\lambda}(r, \tilde{\Omega}) \approx \left\{ I_{\lambda}^{BS}(r, \tilde{\Omega}) + \frac{\rho_{\text{soil}}(\lambda)}{1 - \rho_{\text{soil}}(\lambda) \cdot R_{S}(\lambda)} \cdot T_{BS}(\lambda) \cdot I_{\lambda}^{S}(r, \tilde{\Omega}) \right\} \cdot F_{\lambda}(\tilde{\Omega}_0),
\]

for reflectance (albedo),

\[
R(\lambda) = R_{BS}(\lambda) + \frac{\rho_{\text{soil}}(\lambda)}{1 - \rho_{\text{soil}}(\lambda) \cdot R_{S}(\lambda)} \cdot T_{BS}(\lambda) \cdot T_{S}(\lambda),
\]

for absorbance,
\[
A(\lambda) = A_{BS}(\lambda) + \frac{\rho_{\text{soil}}(\lambda)}{1 - \rho_{\text{soil}}(\lambda) \cdot R_s(\lambda)} \cdot T_{BS}(\lambda) \cdot A_S(\lambda), \tag{21}
\]

and transmittance
\[
T(\lambda) = T_{BS}(\lambda) + \frac{\rho_{\text{soil}}(\lambda)}{1 - \rho_{\text{soil}}(\lambda) \cdot R_s(\lambda)} \cdot T_{BS}(\lambda) \cdot R_s(\lambda), \tag{22}
\]

where \( R_i(\lambda), T_i(\lambda), A_i(\lambda) \) are the hemispherical reflectance, transmittance and absorbance for corresponding ‘i’ problem (i=’S’ or ‘BS’ problem) (cf. Section 4).

3 Transfer Equation for the Mean Intensity

The motivation to find mean intensity of solar radiation interacting with vegetation canopy is simple: sensors aboard satellite platforms measure the mean field emanating from smallest area to be resolved, a pixel. One possible modeling approach to this problem is to generate a set of stochastic realizations of vegetation canopies, solve the classical stochastic RTE [Eq. (10)] and average the solutions. A highly desirable alternative to this computationally intensive process is to derive a transport equation for the mean field directly. This problem has been a highly active research field in recent years (Pomraning, 1995). As mentioned previously the closed system of equations,
describing mean intensity of radiation was developed for broken clouds by Vainikko (1973a, 1973b) and it can be applied to vegetation canopies.

In this section, we will use Vainikko’s approach to derive the equations for mean radiation intensity in a vegetation canopy. We are interested in two kinds of mean intensities: (1) the mean intensity over vegetated area, at the level \( z \in [0, H] \),

\[
U(z, \tilde{\Omega}) = \lim_{R \to \infty} \frac{1}{\text{Mes}(S_R \cap T_z)} \iint_{S_R \cap T_z} I(x, y, z, \tilde{\Omega}) \, dx \, dy
\]  

where \( T_z \) is part of horizontal plane \( z \) covered by vegetation and \( \text{Mes}(S_R \cap T_z) \) means area of plane \( z \) covered by vegetation which is inside a circle \( S_R \) defined at the same plane; and (2) the mean intensity over total space, at the level \( z \in [0, H] \),

\[
\bar{I}(z, \tilde{\Omega}) = \lim_{R \to \infty} \frac{1}{\pi R^2} \iint_{S_R} I(x, y, z, \tilde{\Omega}) \, dx \, dy .
\]

The following important property of stochastic intensity \( I(\vec{r}, \Omega) \) is valid

\[
U(z, \tilde{\Omega}) = \lim_{R \to \infty} \frac{1}{\text{Mes}(S_R \cap T_z \cap T_z(x_i, y_i))} \iint_{S_R \cap T_z \cap T_z(x_i, y_i)} I(x, y, z, \tilde{\Omega}) \, dx \, dy,
\]

which simply means that manifold \( T_z \cap T_z(x_i, y_i) \) contains the same percentage of vegetation as the total manifold \( T_z \). This is the so called assumption of “local chaotisity and global order” (Vainikko, 1973a).

The procedure to derive the transfer equation for mean intensity from the classical approach is as follows: First, the classical stochastic transfer equation is integrated from boundaries \( (z = 0 \) and \( z = H) \) to the some inner point \( z \in [0, H] \), in order to obtain a linear integral equation, which still describes a particular random realization of vegetated
elements. Second, the transfer equation is averaged over the whole plane \( z \) to derive a formula for \( \bar{I}(z, \Omega) \), which is the mean intensity over the whole horizontal plane. The equation for \( \bar{I}(z, \Omega) \) depends on and is expressed through \( U(z, \Omega) \),

\[
\bar{I}(z, \Omega) = f[U(z, \Omega), \ldots].
\]

Third, the transfer equation is averaged over part of a horizontal plane \( z \), which is covered by vegetation \( \chi(\vec{r}) = 1 \), in order to derive the system for unknown \( U(z, \Omega) \), mean radiance over the vegetated portion of the plane.

The averaging procedure, as a general rule, results in equations, which contain some parameters descriptive of characteristic moments of the media (correlation function and mean value). The equations for \( \bar{I}(z, \Omega) \) and \( U(z, \Omega) \) depend on the following mean statistical functions, which must be obtained through corresponding procedure of modeling the vegetation,

\[
q(z, \xi, \tilde{\Omega}) = \lim_{R \to \infty} \frac{\text{Mes} \left\{ S_R \cap T_z \cap T_\xi \left[ \frac{\Omega_z}{\Omega_z} (z - \xi), \frac{\Omega_z}{\Omega_z} (z - \xi) \right] \right\}}{\pi R^2},
\]

which is the probability of finding simultaneously vegetation elements at locations \( M_1(x, y, z) \) and \( M_2(\mu, \eta, \zeta) \) along the direction \( \tilde{\Omega} \). In the above,

\[
\text{Mes} \left\{ S_R \cap T_z \cap T_\xi \left[ \frac{\Omega_z}{\Omega_z} (z - \xi), \frac{\Omega_z}{\Omega_z} (z - \xi) \right] \right\},
\]
indicates the part of vegetation located on a horizontal plane at depth $z$ that overlaps with vegetation located on a horizontal plane at depth $\xi$, if the two planes are moved towards one another along $\bar{\Omega}$, while keeping them parallel until they collapse. Further,

$$p(z) = \lim_{R \to \infty} \frac{\text{Mes}\{S_R \cap T_z\}}{\pi R^2} \quad (26)$$

is the probability of finding foliage elements at depth $z$, or HDV [as defined earlier at Eq. (4)]. And,

$$K(z, \xi, \bar{\Omega}) = \frac{q(z, \xi, \bar{\Omega})}{p(z)} \quad (27)$$

is the conditional probability of finding a vegetation element at point $M_2(\mu, \eta, \xi)$, which is located along the direction $\Omega$ from $M_1(x, y, z)$ in the plane $\xi$ given $M_1(x, y, z) \in$ vegetation.

The detailed procedure to derive the mean RTE is as follows [we follow the procedure of Vainikko (1973a)]. We start with Eq. (10) and rewrite it in the form,

$$\begin{align*}
\Omega_x \frac{\partial I(x, y, z, \bar{\Omega})}{\partial x} + \Omega_y \frac{\partial I(x, y, z, \bar{\Omega})}{\partial y} + \Omega_z \frac{\partial I(x, y, z, \bar{\Omega})}{\partial z} &= g(x, y, z, \bar{\Omega}),
\end{align*} \quad (28)$$

where the following notation was used

$$g(x, y, z, \bar{\Omega}) \equiv -\chi(\bar{r})\sigma(\bar{\Omega})I(\bar{r}, \bar{\Omega}) + \chi(\bar{r}) \int_{4\pi} \sigma_s(\bar{\Omega}' \rightarrow \bar{\Omega}) I(\bar{r}, \bar{\Omega}') d\bar{\Omega}', \quad (29)$$

Integration of Eq. (28) from boundaries ($z=0$ and $z=H$) to some inner point $\bar{r} \sim (x, y, z), z \in [0, H]$ along the direction $\bar{\Omega}$ (Fig. 2) results in the next system of equations
\[
\begin{aligned}
I(x, y, z, \Omega) &= I(x, y, 0, \Omega) + \frac{1}{\mu(\Omega)} \int_0^z g(x + \frac{\Omega_x}{\Omega_z}(z - \xi), y + \frac{\Omega_y}{\Omega_z}(z - \xi), \xi, \Omega) \, d\xi, \quad \mu(\Omega) < 0, \\
I(x, y, z, \Omega) &= I(x, y, H, \Omega) + \frac{1}{\mu(\Omega)} \int_z^H g(x + \frac{\Omega_x}{\Omega_z}(z - \xi), y + \frac{\Omega_y}{\Omega_z}(z - \xi), \xi, \Omega) \, d\xi, \quad \mu(\Omega) > 0.
\end{aligned}
\] (30)

Inserting formulas for function \(g(x, y, z, \tilde{\Theta})\) \([\text{Eq. (29)}]\) into Eq. (30), we obtain the following system:

\[
\begin{aligned}
I(x, y, z, \Omega) + \frac{1}{\mu(\Omega)} \int_0^z \chi(...) \sigma(\Omega) I(..., \Omega) \, d\xi &= \frac{1}{\mu(\Omega)} \int_0^z \chi(...) \sigma_\sigma(\tilde{\Omega} \rightarrow \tilde{\Theta}) I(..., \tilde{\Omega}) \, d\tilde{\Theta} \\
+ I(x, y, 0, \tilde{\Theta}), \quad \Omega_z < 0, \\
I(x, y, z, \Omega) + \frac{1}{\mu(\Omega)} \int_z^H \chi(...) \sigma(\Omega) I(..., \Omega) \, d\xi &= \frac{1}{\mu(\Omega)} \int_z^H \chi(...) \sigma(\tilde{\Omega} \rightarrow \tilde{\Theta}) I(..., \tilde{\Omega}) \, d\tilde{\Theta} \\
+ I(x, y, H, \tilde{\Theta}), \quad \Omega_z > 0.
\end{aligned}
\] (31)

where for simplicity the following short cut was introduced

\[
\begin{aligned}
... &\equiv x + \frac{\Omega_x}{\Omega_z}(z - \xi), y + \frac{\Omega_y}{\Omega_z}(z - \xi), \xi
\end{aligned}
\] (32)

The next step is to average Eq. (31) over the horizontal plane \(z, \, z \in [0, H]\). The main problem is to average item \(\chi(...) I(..., \Omega)\). It can be done rather straightforwardly after shifting manifold \(T_\xi\) by vector \(\left\{ \frac{\Omega_x}{\Omega_z}(z - \xi), \frac{\Omega_y}{\Omega_z}(z - \xi) \right\}\), namely,

\[
\frac{1}{\pi R^2} \iint_{S_R} \chi(...) I(..., \Omega) \, dx \, dy = \frac{1}{\pi R^2} \int_{S_R \cap T_\xi \left[ \frac{\Omega_x}{\Omega_z}(z - \xi), \frac{\Omega_y}{\Omega_z}(z - \xi) \right]} \chi(...) I(..., \Omega) \, dx \, dy =
\]

\[
= \frac{\text{Mes}(S_R \cap T_\xi)}{\pi R^2} \frac{1}{\text{Mes}(S_R \cap T_\xi)} \iint_{S_R \cap T_\xi} I(x', y', \xi', \Omega) \, dx' \, dy',
\]

where

\[
\text{Mes}(S_R \cap T_\xi)
\]
\[ S'_r = S_r \left[ \frac{\Omega_x(z - \xi)}{\Omega_z} \frac{\Omega_y(z - \xi)}{\Omega_z} \right]. \]

If we recall Eq. (26),

\[
p(z) = \lim_{R \to \infty} \frac{\text{Mes}\{S_r \cap T_s\}}{\pi R^2},
\]

we obtain the next limit

\[
\lim_{R \to \infty} \frac{1}{\pi R^2} \int \chi(...) I(..., \Omega) d\xi = p(\xi) \cdot U(\xi, \Omega). \tag{33}
\]

Keeping in mind Eq. (33) while averaging Eq. (31) over the entire plane \(z\), we finally have

\[
\begin{cases}
\int (z, \Omega) + \frac{1}{\mu(\Omega)} \int_0^z \sigma(\Omega) p(\xi) U(\xi, \Omega) d\xi = \frac{1}{\mu(\Omega)} \int_0^z d\xi p(\xi) \int_{\Omega} \sigma(\Omega') \to \Omega U(\xi, \Omega') d\Omega' \\
+ \bar{I}(x, y, 0, \Omega), \quad \Omega_z < 0,
\end{cases}
\]

\[
\begin{cases}
\int (z, \Omega) + \frac{1}{\mu(\Omega)} \int_0^z \sigma(\Omega) p(\xi) U(\xi, \Omega) d\xi = \frac{1}{\mu(\Omega)} \int_0^z d\xi p(\xi) \int_{\Omega} \sigma(\Omega') \to \Omega U(\xi, \Omega') d\Omega' \\
+ \bar{I}(x, y, \bar{H}, \Omega), \quad \Omega_z > 0.
\end{cases}
\tag{34}
\]

The last step is to average the system of Eq. (31) over the portion of horizontal plane \(z\), \(z \in [0, H]\), covered by vegetation, \(T_z\). Now we need to obtain the value of \(\chi(...) I(..., \Omega)\) after averaging over \(T_z\),

\[
\frac{1}{\text{Mes}(S^\prime_r \cap T^\prime_z)} \int \chi(...) I(..., \Omega) d\xi dy = \frac{1}{\text{Mes}(S^\prime_r \cap T^\prime_z)} \int \chi(...) I(..., \Omega) d\xi dy = \frac{\text{Mes}(S^\prime_r \cap T^\prime z \cap T^\prime_z)}{\text{Mes}(S^\prime_r \cap T^\prime_z) / \pi R^2} \int \chi(...) I(..., \Omega) d\xi dy',
\]

\[
= \frac{\text{Mes}(S^\prime_r \cap T^\prime z \cap T^\prime_z)}{\pi R^2 \text{Mes}(S^\prime_r \cap T^\prime_z)} \int \chi(...) I(..., \Omega) d\xi dy',
\]

\[
= \frac{\text{Mes}(S^\prime_r \cap T^\prime z \cap T^\prime_z)}{\pi R^2 \text{Mes}(S^\prime_r \cap T^\prime_z)} \int \chi(...) I(..., \Omega) d\xi dy'.
\]
where \( T'_z = T_z \left\{ \frac{\Omega_x}{\Omega_z}, (z - \xi), \frac{\Omega_y}{\Omega_z}(z - \xi) \right\} \). Taking into account,

\[
\text{Mes} \left( S'_{R} \cap T'_z \cap T_z \right) = \text{Mes} \left\{ S_R \cap T'_z \cap T_z \left[ \frac{\Omega_x}{\Omega_z}, (z - \xi), \frac{\Omega_y}{\Omega_z}(z - \xi) \right] \right\},
\]

we obtain

\[
\lim_{R \to \infty} \frac{1}{\text{Mes} \left( S_R \cap T_z \right)} \int \int \chi(...)(...,\Omega) dxdy = K(z, \xi, \Omega)U(\xi, \Omega).
\]

(35)

Keeping in mind Eq. (35) while averaging Eq. (31) over \( T_z \) we finally have

\[
\begin{cases}
U(z, \Omega) + \frac{1}{\mu(\Omega)} \int_0^\infty \sigma(\Omega) K(z, \xi, \Omega) U(\xi, \Omega) d\xi = \frac{1}{\mu(\Omega)} \int_0^\infty d\xi K(z, \xi, \Omega) \\
\times \int \sigma_s(\tilde{\Omega}' \rightarrow \tilde{\Omega}) U(\xi, \tilde{\Omega}') d\tilde{\Omega}' + \bar{I}(x, y, 0, \tilde{\Omega}), \quad \Omega_z < 0,
\end{cases}
\]

\[
\begin{cases}
U(z, \Omega) + \frac{1}{\mu(\Omega)} \int_0^1 \sigma(\Omega) K(z, \xi, \Omega) U(\xi, \Omega) d\xi = \frac{1}{\mu(\Omega)} \int_0^1 d\xi K(z, \xi, \Omega) \\
\times \int \sigma_s(\tilde{\Omega}' \rightarrow \tilde{\Omega}) U(\xi, \tilde{\Omega}') d\tilde{\Omega}' + \bar{I}(x, y, H, \tilde{\Omega}), \quad \Omega_z > 0.
\end{cases}
\]

(36)

The systems (34) and (36) together form a complete set of equations to determine mean intensity of radiation in a vegetation canopy.

### 3.1 Direct and Diffuse Components of \( U(z, \tilde{\Omega}) \)

In order to be consistent with boundary conditions [Eq. (13)], the function \( U(z, \Omega) \) can be represented as follows
\[ U(z, \Omega) = C_\lambda U_\delta(z) \delta(\Omega - \Omega_0) + F_\lambda(\Omega_0) U_d(z, \Omega) \equiv \]
\[ = F_\lambda(\Omega_0) \left[ \frac{f_{\text{dir}}(\lambda)}{\mu(\Omega_0)} U_\delta(z) \delta(\Omega - \Omega_0) + U_d(z, \Omega) \right]. \tag{37} \]

where \( U_\delta(z) \) is the direct component and \( U_d(z, \Omega) \) is the diffuse component of total mean intensity over the vegetated area. Inserting Eq. (37) into Eq. (35) we obtain equations for these two functions. Equation for \( U_\delta(z) \), the direct component is

\[ U_\delta(z) + \frac{\sigma(\Omega_0)}{\mu(\Omega_0)} \int_0^z K(z, \xi, \Omega_0) U_\delta(\xi) d\xi = 1. \tag{38} \]

The system of equations for \( U_d(z, \Omega) \), the diffuse components is

\[
\begin{cases}
U_d(z, \Omega) + \frac{\sigma(\Omega)}{\mu(\Omega)} \int_0^z K(z, \xi, \Omega)U_d(\xi, \Omega) d\xi = \frac{1}{\mu(\Omega)} \int_0^z K(z, \xi, \Omega) S(\xi, \Omega) d\xi \\
+ U_{d0}^\prime(z, \Omega, \Omega_0), \quad \mu < 0, \\
U_d(z, \Omega) + \frac{\sigma(\Omega)}{\mu(\Omega)} \int_z^H K(z, \xi, \Omega)U_d(\xi, \Omega) d\xi = \frac{1}{\mu(\Omega)} \int_z^H K(z, \xi, \Omega) S(\xi, \Omega) d\xi \\
+ U_{dH}^\prime(z, \Omega, \Omega_0), \quad \mu > 0, \tag{39}
\end{cases}
\]

where:

\[ S(\xi, \Omega) = \int_{4\pi} \sigma_\delta(\Omega \rightarrow \Omega') U_d(\xi, \Omega') d\Omega', \]

\[ U_{d0}^\prime(z, \Omega, \Omega_0) = \frac{f_{\text{dir}}(\lambda) \sigma_\delta(\Omega_0 \rightarrow \Omega)}{\mu(\Omega) \mu(\Omega_0)} \int_0^z K(z, \xi, \Omega) U_\delta(\xi) d\xi + \left[ 1 - f_{\text{dir}}(\lambda) \right] I_d(\Omega, \Omega_0), \]

\[ U_{dH}^\prime(z, \Omega, \Omega_0) = \frac{\sigma_\delta(\Omega_0 \rightarrow \Omega)}{\mu(\Omega) \mu(\Omega_0)} \int_z^H K(z, \xi, \Omega) U_\delta(\xi) d\xi + I_{\text{H}}(\Omega). \]

### 3.2 Direct and Diffuse Components of \( \bar{I}(z, \Omega) \)

Similar to the case of \( U(z, \Omega) \), the function \( \bar{I}(z, \Omega) \) should be consistent with the boundary condition [Eq.13] and can be represented as follows
\[ \bar{I}(z, \Omega) = C_s I_\delta(z) \delta(\Omega - \Omega_0) + F_\chi(\Omega_0) \bar{I}_d(z, \Omega) \equiv \]
\[ \equiv F_\chi(\Omega_0) \left[ \int f_{\text{dir}}(\lambda) I_\delta(z) \delta(\Omega - \Omega_0) + I_d(z, \Omega) \right], \tag{40} \]

where \( \bar{I}_\delta(z) \) is the direct component and \( \bar{I}_d(z, \Omega) \) is the diffuse component of total mean intensity over the total canopy space. Combining Eq. (34) into Eq. (40) we obtain equations for these functions. The equation for the direct component, \( \bar{I}_\delta(z) \), is

\[ \bar{I}_\delta(z) = 1 - \frac{\sigma(\Omega_0)}{\mu(\Omega_0)} \int_0^z p(\xi) U_d(\xi) d\xi, \tag{41} \]

The system of equations for \( \bar{I}_d(z, \Omega) \), the diffuse component is

\[
\begin{align*}
\bar{I}_d(z, \Omega) &= -\frac{\sigma(\Omega)}{\mu(\Omega)} \int_0^z p(\xi) U_d(\xi, \Omega) d\xi + \frac{1}{\mu(\Omega)} \int_0^z p(\xi) S(\xi, \Omega) + I_d^1(z, \Omega, \Omega_0), \quad \mu < 0, \\
\bar{I}_d(z, \Omega) &= -\frac{\sigma(\Omega)}{\mu(\Omega)} \int_0^z p(\xi) U_d(\xi, \Omega) d\xi + \frac{1}{\mu(\Omega)} \int_0^z p(\xi) S(\xi, \Omega) + I_d^1(z, \Omega, \Omega_0), \quad \mu > 0,
\end{align*}
\] where \( S(\xi, \Omega) = \int \sigma(\Omega' \rightarrow \Omega) U_d(\xi, \Omega') d\Omega' \). \tag{42}

\[
\begin{align*}
I_d^1(z, \Omega_0, \Omega_0) &= \int \frac{f_{\text{dir}}(\lambda) \sigma(\Omega_0 \rightarrow \Omega)}{\mu(\Omega_0) \mu(\Omega)} \int_0^z p(\xi) U_d(\xi) d\xi + \left[1 - f_{\text{dir}}(\lambda)\right] d(\Omega, \Omega_0), \quad \mu < 0, \\
I_d^1(z, \Omega_0, \Omega_0) &= \int \frac{f_{\text{dir}}(\lambda) \sigma(\Omega_0 \rightarrow \Omega)}{\mu(\Omega_0) \mu(\Omega)} \int_0^z p(\xi) U_d(\xi) d\xi + I_d(\Omega, \Omega_0), \quad \mu > 0.
\end{align*}
\]

It is interesting to note that formula for \( \bar{I}(z, \Omega) \) [Eq. (42)] is similar to the system of equations for \( U(z, \Omega) \) [Eq. (39)]. The following important property of the equation for \( U(z, \Omega) \) and \( \bar{I}(z, \Omega) \) is valid: if and only if,

\[ q(z, \xi, \Omega) = p(z) \cdot p(\xi), \tag{43} \]

which results in
\[ K(z, \xi, \Omega) = \frac{q(z, \xi, \Omega)}{p(z)} \equiv \frac{p(z) \cdot p(\xi)}{p(z)} = p(\xi), \]

and then the equations for \( U(z, \Omega) \) and \( I(z, \Omega) \) are identical. This means that the mean intensity over the vegetated area is equal to the mean intensity over the whole space. This corresponds to the turbid medium case where there is no correlation between the distribution of vegetated spaces in the canopy.

Another important note about the system of equations for the mean field is: the equations do not describe the hot spot effect. This is also true for the classical stochastic equation (Eq. 10). Numerical simulations (described later) attest to this. Thus, we use the standard approach to implement the hot spot, that is modify the extinction coefficient, \( \sigma(\hat{\Omega}) \).

4 Vegetation Canopy Energy Balance

The results obtained earlier for the mean intensity are necessary for the analysis of energy fluxes in vegetation canopies. The standard procedure to trace energy input and output to/from the system is to integrate the equation for the mean intensity [Eq. (42)] over canopy space and over all directions. The resulting equation describes the energy conservation law, namely

\begin{align*}
A + R + [1 - p_{\text{soil}}(\lambda)] \cdot T &= 1, \quad \text{(general problem)}, \quad \text{(44 a)} \\
A_i + R_i + T_i &= 1, \quad (i = \text{BS or S problem}), \quad \text{(44 b)}
\end{align*}
where $A$ is absorptance, $R$ is reflectance and $T$ is transmittance for general problem, defined by Eq. (10) and Eq. (13) or Eq. (39) and Eq (42); $A_i$, $R_i$, $T_i$ represent the same, but for BS and S problems. As mentioned earlier, solution of the general problem can be expressed through one of BS and S problems [Eq. (19) through Eq. (22)], so we need only to give the final expressions for $A_i$, $R_i$, $T_i$. In the case of Black Soil problem these are,

$$A_{BS}(\lambda) = (1 - \omega_\lambda) \left\{ \int_0^H d\xi \int \frac{d\Omega}{4\pi} p(\xi) \sigma(\Omega) U_d(\xi, \Omega) + \int_0^H \frac{f_{\text{dir}}(\lambda) \sigma(\Omega_0)}{\mu(\Omega_0)} \int_0^H p(\xi) U_d(\xi, \Omega) d\xi \right\},$$

$$R_{BS}(\lambda) = \int_{2\pi^-} I_d(0, \Omega) \mu(\Omega) d\Omega,$$

$$T_{BS}(\lambda) = \int_{2\pi^+} I_d(H, \Omega) \mu(\Omega) d\Omega + f_{\text{dir}}(\lambda) I_0(H),$$

and for the Soil problem,

$$A_S(\lambda) = (1 - \omega_\lambda) \left\{ \int_0^H d\xi \int \frac{d\Omega}{4\pi} p(\xi) \sigma(\Omega) U_d(\xi, \Omega) \right\},$$

$$R_S(\lambda) = \int_{2\pi^-} I_d(H, \Omega) \mu(\Omega) d\Omega,$$

$$T_S(\lambda) = \int_{2\pi^+} I_d(0, \Omega) \mu(\Omega) d\Omega.$$

Note that absorptance for both problems is different from that of the turbid medium case, that is, the absorptance is expressed not through mean intensity over the total space, $I(z, \Omega)$, but through mean intensity over the vegetated area, $U(z, \Omega)$. This is due to the
fact that energy can be absorbed only by foliage elements, not by voids between leaves (or
between trees). One can compare this result with that for the turbid medium, taking the
BS problem as an example,

\[ A = \int d\xi \int d\Omega \sigma_a(\Omega) \tilde{I}(\xi, \Omega) = \left[ I - \alpha(\lambda) \right] \int d\xi \int d\Omega \sigma_a(\Omega) \tilde{I}(\xi, \Omega) = \]

\[ = \left[ 1 - \alpha(\lambda) \right] \left\{ \int d\xi \int d\Omega \sigma_a(\Omega) \tilde{I}_d(\xi, \Omega) + \frac{f_{\text{dir}}(\lambda) \cdot \sigma(\tilde{\Omega}_0)}{\mu(\tilde{\Omega}_0)} \int_0^H \tilde{I}_d(\xi, \Omega) d\xi \cdot \right\}. \]

(47)

The new formula for absorptance can be derived starting with its physical definition

\[ A \equiv \int \int I(r, \tilde{\Omega}) \chi(r) \sigma(\tilde{\Omega}) d\tilde{r} d\tilde{\Omega}. \]

(48)

Assuming that the incident radiant energy is normalized to unity,

\[ \int_{2\pi} I(z = 0, x, y, \tilde{\Omega}) \mu(\tilde{\Omega}) \mid dxdy = \pi R^2, \]

the correct expression for absorptance is,

\[ A = \frac{1}{\pi R^2} \int \int I(r, \tilde{\Omega}) \chi(r) \sigma(\tilde{\Omega}) d\tilde{r} d\tilde{\Omega} = \frac{1}{\pi R^2} \int_0^H dz \int d\tilde{\Omega} \int \chi(r) \sigma(\tilde{\Omega}) I(r, \tilde{\Omega}) dxdy = \]

\[ = \int_0^H dz \ \Omega p(\ ) (\Omega \ U( , \Omega) \)

\[ \lim_{R \to 0} \frac{1}{\pi R^2} \int \int I(x, y, z, \tilde{\Omega}) \chi(x, y, z) dxdy = p(z) U(z, \tilde{\Omega}). \]
Finally, the new formulation for absorbance in the general discontinuous case collapses to
the standard turbid medium definition if,

\[ K(z, \xi, \Omega) \equiv \frac{q(z, \xi, \Omega)}{p(z)} = \frac{p(z) \cdot p(\xi)}{p(z)} = p(\xi). \]

This expresses the absence of correlation between vegetated elements located at \( z \) and \( \xi \).

5 Numerical Solution of Mean RTE

In order to solve the system of integral equations for mean intensities \( U(z, \Omega) \), [Eqs. (38)
and (39)] and \( I(z, \Omega) \), [Eqs. (41) and (42)], a model of vegetation canopy structure is
required together with a numerical scheme for solution of the corresponding transfer
equations. Important variables in the equations for mean intensities are functions \( p(z) \) and
\( K(z, \xi, \Omega) \), which can be obtained from a model of canopy structure. Note that \( p(z) \) is the
probability of finding vegetated area in a horizontal plane at depth \( z \in [0; H] \); and
\( K(z, \xi, \Omega) \) is the conditional probability of the presence of vegetated areas in planes \( z \) and
\( \xi \), where \( z, \xi \in [0; H] \). We used a simple model of a vegetation canopy by representing
the plants or trees as parallelepipeds distributed on the ground with probability \( p(H) \).

Further, they do not overlap and all have the same dimensions (height, width and depth),
and \( p(z) = \text{const} = p(H) \) is equal to the portion of the plane covered by vegetation.

Function \( K(z, \xi, \Omega) \) was calculated by implementing the definition of \( K(z, \xi, \Omega) \) [Eq. (27)].
In the system of integral equations for \( \bar{I}(z, \Omega) \) and \( U(z, \Omega) \) [Eqs. (49) and (42)], one needs to solve only the system for \( U(z, \Omega) \). The evaluation of \( \bar{I}(z, \Omega) \) is a straightforward numerical integration of \( U(z, \Omega) \). In order to solve the system for \( U(z, \Omega) \), the method of successive orders of scattering approximations (SOSA) was used (Myneni et al., 1987).

The n-th approximation to the solution is given by

\[
U_0^n(z, \Omega) = J_1(z, \Omega) + J_2(z, \Omega) + \ldots + J_n(z, \Omega).
\]

The functions \( J_k(z, \Omega), k = 1, 2, ..., n \) are the solutions of the system of two independent equations:

\[
J_k(z, \Omega) + \frac{\sigma(\Omega)}{\mu(\Omega)} \int_0^z K(z, \xi, \Omega) J_k(\xi, \Omega) d\xi = R_{k-1}(z, \Omega), \quad \mu < 0, \tag{49}
\]

\[
J_k(z, \Omega) + \frac{\sigma(\Omega)}{\mu(\Omega)} \int_z^H K(z, \xi, \Omega) J_k(\xi, \Omega) d\xi = R_{k-1}(z, \Omega), \quad \mu > 0, \tag{50}
\]

where

\[
R_k(z, \Omega) = \frac{1}{\mu(\Omega)} \int_0^z K(z, \xi, \Omega) S_k(\xi, \Omega) d\Omega + \frac{f_{\text{dir}}(\lambda) \sigma_s(\Omega_0 \rightarrow \Omega)}{\mu(\Omega) \mu(\Omega_0)} \int_0^z K(z, \xi, \Omega) U_s(\xi, \Omega) d\xi
\]

\[
+ [1 - f_{\text{dir}}(\lambda)] d(\Omega, \Omega_0), \quad \mu < 0, \quad \text{when } k \geq 1,
\]

\[
R_k(z, \Omega) = \frac{1}{\mu(\Omega)} \int_z^H K(z, \xi, \Omega) S_k(\xi, \Omega) d\Omega + \frac{f_{\text{dir}}(\lambda) \sigma_s(\Omega_0 \rightarrow \Omega)}{\mu(\Omega) \mu(\Omega_0)} \int_z^H K(z, \xi, \Omega) U_s(\xi, \Omega) d\xi
\]

\[
+ I_1(\Omega, \Omega_0), \quad \mu > 0, \quad \text{when } k \geq 1,
\]

and the source function \( S(z, \Omega) \) is,
\[ S_k(z, \Omega) = \int_{4\pi} \sigma_s(\Omega' \to \Omega) \, J_k(z, \Omega') \, d\Omega'. \]

Note that in order to calculate \( R_0(z, \Omega) \) (\( k = 0 \)), initially \( S(z, \Omega) = 0 \). The algorithm to solve the system of equations for \( U(z, \Omega) \) is as follows: (1) Find \( U_0(z, \Omega) \) from the corresponding Volterra equation [Eq. (38)]; (2) Set \( S(z, \Omega) = 0 \) and evaluate \( R_0(z, \Omega) \); (3) Solve the Volterra equations [Eq. (49) and Eq.(50)] with \( R_0(z, \Omega) \) and find \( J_1(z, \Omega) \); (4) Evaluate \( S_1(z, \Omega) = \int_{4\pi} \sigma_s(\Omega' \to \Omega) \, J_1(z, \Omega') \, d\Omega' \) with \( J_1(z, \Omega) \); (5) Evaluate \( R_1(z, \Omega) \); (6) Calculate \( J_2(z, \Omega) \); (7) Repeat the following until \( \| J_n(z, \Omega) \| \leq \epsilon \) : (a) Evaluate \( S_k(z, \Omega) \); (b) Calculate \( R_k(z, \Omega) \); (c) Calculate \( J_{k+1}(z, \Omega) \).

The numerical method used to solve the basic equations is as follows. We start with the parametric Volterra equation,

\[ U(z, \Omega) + \frac{\sigma(\Omega)}{|\mu(\Omega)|} \int_0^z K(z, \xi, \Omega) \, U(\xi, \Omega) \, d\xi = F(z, \Omega). \]  

Here \( \Omega \) is a parameter of the equation. The corresponding discretization scheme is

\[ U(k, \Omega) + \frac{\sigma(\Omega)}{|\mu(\Omega)|} \sum_{j=1}^{j=k} W_{k,j} K(k, j, \Omega) \, U(j, \Omega) = F(k, \Omega), \]  

where \( W_{k,j} \) is the weight, which depends on the numerical scheme used for approximating the integral. Then,

\[ U(1, \Omega) = F(1, \Omega), \]  

when \( k = 1 \), and when \( k \in [2, N_z + 1] \),
\[ \begin{align*}
U(k, \Omega) + \frac{\sigma(\Omega)}{\mu(\Omega)} W_{k,k}K(k, k, \Omega)U(k, \Omega) = F(k, \Omega) - \frac{\sigma(\Omega)}{\mu(\Omega)} \sum_{j=1}^{j=k-1} W_{k,j}K(k, j, \Omega) \ U(j, \Omega),
\end{align*} \]

\[ \Rightarrow \quad U(k, \Omega) = \frac{F(k, \Omega) - \frac{\sigma(\Omega)}{\mu(\Omega)} \sum_{j=1}^{j=k-1} W_{k,j}K(k, j, \Omega) \ U(j, \Omega)}{1 + \frac{\sigma(\Omega)}{\mu(\Omega)} W_{k,k}K(k, k, \Omega)}. \quad (53) \]

Another important method used in this algorithm is the method of \( S_n \) quadratures of Carlson (Bass et al., 1986) to evaluate angular integrals. This scheme belongs to the method of Gauss quadratures. The quadrature is built as follows. The octant is divided into \( \frac{n \cdot (n+2)}{8} \) parts of equal area, \( w_0 = \frac{4\pi}{n \cdot (n+2)} \) using latitudes, defined as \( \mu = \mu_{\ell + \frac{1}{2}}, \ \ell = 0, 1, \ldots, \frac{n}{2} \) and longitudes, defined as \( \varphi = \varphi_{\ell, m + \frac{1}{2}}, \ m = 0, 1, \ldots, \frac{n}{2} - \ell + 1 \). The coordinates of the boundaries of each layer are:

\[ \mu_{\ell + \frac{1}{2}} = 1 - \left( \frac{(n - 2\ell + 2) \cdot [n - 2(\ell - 1 \pm 1)]}{n \cdot (n+2)} \right), \]

and the coordinates of the centers of layers are

\[ \mu_{\ell} = 1 - \left[ \frac{n - 2\ell + 2}{n \cdot (n+2)} \right]^2. \]

The nodes of quadratures are

\[ \begin{cases} 
\mu_{\ell} = \mu_{\ell} + f \cdot \mu_{\ell-1}, & \ell = 0, 1, \ldots, \frac{n}{2}, \\
\varphi_{\ell, m} = \frac{\pi}{2} \left[ \frac{2m - 1}{n - 2\ell + 2} A_n + \frac{1}{2} (1 - A_n) \right], & m = 1, 2, \ldots, \frac{n}{2} - \ell + 1, 
\end{cases} \]

and the coefficient \( f \) and \( A_n \) are determined to minimize integral.
Generally, about 30 iterations are sufficient to obtain relative accuracy of $10^{-3}$. The physical interpretation of the method of successive orders is obvious: the function $J_k(z, \Omega)$ is the mean radiance of photons scattered $k$ times. The rate of convergence of this method, $\rho_c$ has been defined by Vladimirov (1963), Marchuk and Lebedev (1971) as

$$\|I - I_0\| \leq \rho_c = (1 - \exp(-k_0 \cdot H)) \cdot \eta \cdot n,$$

(54)

where $k_0$ is a certain coefficient and

$$\eta = \sup_{\Omega_0 < H \Omega \neq H} \frac{\sigma_5(\Omega_0 \rightarrow \Omega)}{\sigma(z, \Omega)}.$$

(55)

From Eq. (44) it follows that SOSA should be used in the case of small optical depth of the layer or in case of small $\eta$. If $\eta = 1$ and the optical depth is large, the method becomes tedious.

6 Evaluation of the Model

To illustrate the characteristics of the mean RTE model described here, numerical results of calculations of important quantities, such as directional reflectances (BRF) in the principal plane, energetic quantities [absorbtance, transmittance, reflectance (DHR/BHR)] are presented in this section. The input variables of Mean RTE model are: (1) solar illumination variables: solar angle $\Omega_0$ and the ratio of direct to total incident flux; (2) canopy geometry, including height $H$, and horizontal dimensions of individual vegetation
units (trees, shrubs), d; (3) statistical moments of the ensemble of vegetation units, namely, functions $p(z)$ and $K(z, \zeta, \Omega)$ defined earlier by Eqs. (26) and (27); (4) characteristics of leaves; density of leaves $u_L(\vec{r})$, leaf normal orientation distribution (uniform, planophile, erectophile, etc.) (Ross, 1975), hemispherical reflectance and transmittance spectra of leaves $r_\circ(\lambda)$, $t_\circ(\lambda)$; and, (5) soil hemispherical reflectance spectra $\rho_{\text{soil}}(\lambda)$. Model outputs are (1) the directional reflectances or the Bi-directional Reflectance Factors (BRF) defined as the surface-leaving radiance, divided by radiance from a conservative Lambertian reflector under monodirectional illumination (Knyazikhin et al., 1998a)

$$\text{BRF} = \frac{I_x(\hat{\omega}_\text{top}, \hat{\omega}, \hat{\omega}_\text{top})}{1/\pi \int I_x(\hat{\omega}_\text{top}, \hat{\omega}, \hat{\omega}_\text{top}) |\hat{\omega} \cdot \hat{n}_\text{top}| d\Omega}, \quad (56)$$

(2) absorbance, transmittance and reflectance. We calculate two types of hemispherical reflectance, BHR and DHR, defined as follows: The Bi-Hemispherical Reflectance (BHR) for non-isotropic incident radiation (both direct and diffuse components) is the ratio of the mean radiant exitance to the incident radiance (Knyazikhin et al., 1998a)

$$\text{BHR} = \frac{\int I_x(\hat{\omega}_\text{top}, \hat{\omega}, \hat{\omega}_\text{top}) |\hat{\omega} \cdot \hat{n}_\text{top}| d\Omega}{\int 2\pi \int I_x(\hat{\omega}_\text{top}, \hat{\omega}, \hat{\omega}_\text{top}) |\hat{\omega} \cdot \hat{n}_\text{top}| d\Omega}, \quad (57)$$

and the Directional Hemispherical Reflectance (DHR) defined similar to BHR, except that the incident radiation has only the direct component. Below, we present results of comparison of the Mean RTE model with other RTE models, Monte Carlo simulations of
the radiation field in a maize canopy and field data from the Jornada PROVE campaign. Issues related to the effect of vegetation clumping on the radiation regime are discussed later.

The three-dimensional dynamic architecture model of maize proposed by España et al. (1999) was utilized for the Monte Carlo simulations. This model allows description of the maize canopy from emergence to male anthesis. Because maize is planted in rows, and for each vegetation unit the leaves are obviously clumped around the stem, the assumption of a random leaf spatial distribution is not valid. The driving parameter of the model is the phenological stage, which is defined by the number of leaves produced since emergence and not totally hidden in the top leafy cone. The model describes the dynamics of dimensions, height, senescence, curvature and insertion angle of the leaves, as well as the temporal evolution of the stem dimensions. Linear equations were developed to describe growth of the leaf size, stem diameter with change of leaf stage and so on. The inputs to the 3D model are: (1) leaf stage (which describes time); (2) plant density, including seeding pattern (row spacing and orientation, plant spacing); (3) leaf area cumulated over the fully developed plant, including leaves that senesced, and (4) final height of the canopy. The maize canopy architecture model was calibrated and validated with three sets of experiments, two of which were performed in Avignon, France (INRA-90, plant density of 12, plant \( \cdot m^{-2} \), canopy measured when the fourteenth leaf appeared; and INRA-97, plant density of 8.5 plant \( \cdot m^{-2} \), measurements performed at two phenological stages, namely, 13 and 17 leaves). The third experiment was performed in
Alpilles, France (Alpilles-97 plant density of \(7 \text{ plant} \cdot \text{m}^{-2}\), measurements performed when the fifteenth leaf was appearing).

The maize canopy was simulated using computer graphic techniques as an assembly of leaves and stems. Six phenological stages of maize were simulated, corresponding to LAI values of 0.25, 0.86, 1.64, 2.34, 3.01 and 6.25 (Table 1). The optical properties of the leaves were determined at three chlorophyll a and b concentrations, \(\text{Cab30}, \text{Cab50}, \text{Cab70}\), which corresponded to 30, 50, 70 \(\mu\text{g} \cdot \text{cm}^{2}\) concentrations of chlorophyll a and b. The optical properties for the case of \(\text{Cab50}\), were used in our validation studies (Table 2).

Dry and wet soils with corresponding optical properties were considered (Baghdadi, 1998). The soil hemispherical reflectances are given in (Table 2). In order to validate the Mean RTE method, results of simulation from a Monte Carlo ray tracing method in a maize canopy were utilized (Baghdadi, 1998; España et al., 1999). A total of three million photons were simulated. The incoming photon beams, corresponding to direct solar illumination, were constrained to have a constant zenith angle of \(45^\circ\), and 150 azimuthal directions were simulated (in intervals of \(2.4^\circ\), where \(0^\circ\) corresponds to the direction perpendicular to the rows). Each of these 150 directions was simulated using twenty thousand photons. There were \(360 \times 90 = 32,400\) viewing directions (steps of \(1^\circ\) along both the zenith and azimuth). The simulations were carried out at 10 wavelengths: 430, 500, 562, 630, 692, 710, 740, 795, 845, 882 in nanometers.
The Mean RTE model was run with the same set of input parameters as the Monte Carlo model. Figures 3, 4 and 5 present results of comparison. We note that not all of the parameters required to parameterize the Mean RTE model were available; for example, ground cover and the horizontal dimensions of maize leaves were not available. Therefore, these parameters were estimated from description of the maize canopy, or in some cases interpolated using available data. Figure 3 shows simulations of the BRF in the principal plane for the case of dry soil, chlorophyll concentration of 50 $\mu g \cdot cm^{-2}$ (Cab 50) at RED (630 nm) and NIR (845 nm) wavelengths, and for three LAI values - low (LAI=0.86), intermediate (LAI=2.34) and high (LAI=6.25). Other parameters are listed in Table 1. The incoming radiation was a monodirectional beam incident at a polar angle of 45°. In Fig. 3, it can be seen that the characteristic shape of BRF changes dramatically from an inverted bowl to a bowl shape. This provides an opportunity to validate the Mean RTE model.

When the total amount of radiation reflected by the vegetation back to the atmosphere is higher than that reflected by bare soil, the BRF will have characteristic bowl shape and the measured quantity, the radiance, is

$$I(\theta, \varphi) \sim \frac{\Phi}{\Omega \cdot A \cdot \cos(\theta)}.$$  

And, this quantity changes with $\theta$ as $\cos(\theta)^{-1}$. When the contribution of the soil to the total amount of reflected energy is higher than contribution of the vegetation, the BRF will have an inverted bowl shape because the incident radiation penetrates deeper into the canopy and is reflected by the soil. Obviously the path in the direction along the nadir is
shorter than along any other direction. So at $\theta = 0$, the energy flux is higher than at along any other direction, and reflectance decreases as $\theta$ increases. The governing formula in this case is

$$I(\theta, \phi) \sim \frac{\Phi \cdot \exp[-\alpha / \cos(\theta)]}{\Omega \cdot A \cdot \cos(\theta)}$$

which results in $I(\theta, \phi)$ decreasing as $\theta \to 90^\circ$.

The next set of plots, Fig. 4, represents validation of the estimated parameters. Here, we used the ‘dry soil’ simulations in order to estimate the full set of parameters, and then ran the Mean RTE model for case of ‘wet soil’, changing only the soil reflectance, which was known from measurements (Baghdadi, 1998). In particular, we interpolated the cover fraction and horizontal dimensions of the maize canopy. For example, from knowledge of LAI and height of the canopy we used the general phenological description of maize (España et al., 1999) in order to find the corresponding phenological stage and than linearly interpolated the cover fraction. The horizontal size of canopy was estimated from fitting the shape of the BRF. Both for training and trainee cases, LAI was 1.64, height 40.7 cm, horizontal size 40 cm, cover fraction 0.35 and chlorophyll concentration 50 $\mu$g $\cdot$ cm$^2$. Figure 5 represents another attempt at validation of the RTE model, in general, and the set of estimated parameters, in particular. The idea was to compare hemispherical reflectances (DHR) of the Monte Carlo method with the mean RTE simulations, under identical set of parameters. We ran the Mean RTE model for the case of ‘wet’ and ‘dry soil’, chlorophyll concentration of 50 $\mu$g $\cdot$ cm$^2$ (Cab 50), at the ten
available wavelengths (Table 1). For the case of dry soil, five values of LAI were used (LAI=0.86; 1.64; 2.34; 3.01; 6.25), and three for the case of wet soil (LAI=1.64; 3.01; 6.25), which results in 50 cases for dry soil and 30 cases for wet soil. The most significant difference (18%) is seen at NIR, where reflectance is high, and in the case of a dense canopy (LAI of 6.25). One source of discrepancy is the numerical scheme used to evaluate DHR from Monte Carlo angular intensities, of the order at least 3%. Importantly, the error in numerical solution of the transfer equation increases at NIR and in dense canopies as follows

$$\Delta I \sim \frac{1}{1 - \alpha(\lambda) \exp(LAI \cdot k)}.$$  

(58)

The performance of the Mean RTE model was also studied in comparison to similar RTE models. We used the 1D model (Shultis and Myneni, 1988) “TWOVEG” and the 3D model “DISORD”, which are numerical methods of solution of one- and three-dimensional radiative transfer equations in plant canopies modeled as turbid media using the discrete ordinates method. Figures 6 and 7 presents the results of comparison between the three models, at two wavelengths RED (645 nm) and NIR (841 nm). The incident radiation was 80% direct and 20% diffuse isotropic sky light, and the solar zenith angle was 30°. The soil hemispherical reflectance was set to 0, so the ‘BS’ problem was studied. Only the case of a homogeneous canopy was considered, so as to include ‘TWOVEG’ also in the comparison. Figure 6 presents the BHR, absorbtance and transmittance versus LAI. All models show the general tendency of absorbtance to increase and transmittance to decrease with increase in LAI. The increase of BHR with
LAI is due to the completely absorbing soil, and as LAI increases, the leaves hide this perfect absorber, and hence the increase in BHR. Figure 7 illustrates dependencies of BHR, absorbtance and transmittance with respect to solar zenith angle at a constant LAI of 5. The general tendency of absorbtance to increase, transmittance to decrease with increase in solar zenith angle is correct, because the pathlength increases as the solar zenith angle increases and the probability of the solar rays to be intercepted also increases. Bihemispherical reflectance also increases with increase in solar zenith angle because at oblique sun angles, more energy is reflected from the boundary and there is correspondingly less penetration into the deeper parts of the canopy. From Figs. 6 and 7, it can be seen that the largest discrepancy between model estimates is for BHR, especially between the Mean RTE and ‘DISORD’ at RED, for absorbtance versus solar zenith angle at both RED and NIR and, for transmittance versus solar zenith angle at RED, which may be attributed to various simplifying approximations used in the numerical solution schemes.

The critical validation of any model is comparison with field data. We utilized data from the Jornada field campaign, distributed by the grassland PROVE (PROtotype Validation Exercise) team (Privette et al., 1999). This experiment took place from April 30 through May 13 of 1997 in Jornada (a large valley near Las Cruces, New Mexico, USA). The area is slowly undergoing a landcover change from a grassland to a shrubland (predominately mesquite). It is a very arid area, so shrubs and grasses are sparse [ground cover is (34.5 ± 2)%, average LAI ~0.5]. The data were from a transitional site (mixed
grassland and shrubs) and consisted of 75% of mesquite and 25% of yucca and morman tea. The measurements were performed on a 26 m tall tower using CIMEL sunphotometer which has 4 channels; we used data from two of these channels: 870 nm and 1020 nm. Related parameters at the Jornada transition site are as follows (for Mesquite): mean height: $1.28 \pm 0.54$ m, LAI: 1.71, leaf reflectance at 870 nm: 0.432, leaf transmittance at 870 nm: 0.395, leaf reflectance at 1020 nm: 0.442, leaf transmittance at 1020 nm: 0.399. For Yucca we have: mean height $0.59 \pm 0.16$ m, LAI: 1.38, leaf reflectance at 870 nm: 0.432, leaf transmittance at 870 nm: 0.107, leaf reflectance at 1020 nm: 0.426, leaf transmittance at 1020 nm: 0.096. Soil reflectance was 0.349 (at 870 nm), and 0.380 (at 1020 nm). Figure 8 presents BRF comparisons (field data and simulated by the Mean RTE) in the principal plane for three values of solar zenith angle ($0^\circ, 40^\circ, 70^\circ$) and at two available wavelengths (1020 nm and 870 nm). Because the data were noisy, it is difficult to perform detailed comparisons, but it appears that the agreement is reasonable. One exception is the comparison for 870 nm at SZA of $40^\circ$, where the simulated BRF is a significant overestimate. There are indications that the data may have a problem in this instance (Privette, private communications): from others plots it is clear that the mean BRF for both channels is approximately the same at a given value of SZA. The plot under consideration is an exception.

Finally, we discuss one important feature of the Mean RTE model, which allows for description of canopy heterogeneity, i.e., mixing of voids with clusters of vegetation elements. The effect of clustering has an important influence on the radiation regime in a
vegetation canopy, for example the hot spot effect and change in the proportions of BHR, absorbtance and transmittance. We already mentioned that heterogeneity leads to a new analytical formula for absorbtance (Sec.4), for which we must use not the mean intensity over all space, but the mean intensity over only the vegetated portion of the canopy space [Eqs. (38) and (39)]. Figures 9 and 10 illustrate the influence of clumping on BHR, absorbtance and transmittance, for the case of a black soil problem, and for a direct to total incident flux ratio of $f_{\text{dir}}(\lambda) = 0.8$. In order to introduce voids, we varied the ground cover parameter and ran the Mean RTE for $p(z)=1.0$ (which corresponds to the turbid medium case), $p(z)=0.75$ and $p(z)=0.5$ (as $p$ decreases, clumping increases). Figure 9 presents the relationship between BHR, absorbtance and transmittance with LAI (SZA is equal to $30^\circ$). Figure 10 presents the same, but with changing solar zenith angle at a constant LAI of 5. The qualitative effect of voids on the radiation regime is similar at NIR and RED wavelengths. From Fig. 9 it is clear that for similar input parameters, in particular LAI, but at different values of ground cover, the BHR and absorbtance decrease as groundcover decrease, and transmittance increases. The same tendency is observed for BHR, absorbtance and transmittance versus solar zenith angle (Fig. 10). The above corresponds to clumping vegetation into clusters, thereby increasing the amount of voids and the probability of solar radiation to penetrate deeper into the canopy which result in increased transmittance and decreased absorbtance. The quantitative effect of clumping is, for example, at LAI=3, SZA=50°, at RED, $\Delta \text{BHR} \equiv |\text{BHR}(p = 1.0) - \text{BHR}(p = 0.5)|/\text{BHR}(p = 1.0) \approx 42.3\%$, similarly, $\Delta \text{Absorbtance} \approx 30\%$ and $\Delta \text{Transmittance} \approx 150\%$.
7 Concluding Remarks

The major goal of this work was to address the problem of accurately describing the influence of discontinuity in vegetation canopies on the radiative regime. The presence of gaps in vegetation canopies introduces corrections to energy fluxes compared to values of fluxes for the homogeneous case and consequently results in large errors in the retrieved biophysical parameters of vegetation such as LAI, FPAR, etc. Among the many current models those, based on geometrical-optical approach are valid for discontinuous vegetation canopies, but they approximate radiative fluxes and multiple scattering of photons; others, based on the classical RT equation, are accurate but applicable only to simple homogeneous cases of crops and grasses.

The proposed approach, based on a statistical formulation of Radiative Transfer Equations, is specially designed to accurately evaluate radiation fluxes in discontinuous vegetation canopies. Special attention was given to deriving analytical results. More specifically, the following major tasks were accomplished. A governing integral equation of the mean field for the transport of monochromatic radiation intensity in spatially heterogeneous canopy has been formulated. The resulting system of integral equations satisfies to the energy conservation law and was solved numerically using the method of successive orders of scattering approximations. The influence of discontinuity on the radiative regime in a vegetation canopy is such, that, a complete description of the
radiation field in the canopy is possible using not only mean radiance over the whole space, but also mean radiance, averaged only over the vegetated part of canopy layer. This approach allows for a correct formulation of absorbtance, which extend its classical definition and in the limiting case of a turbid medium contains the classical definition of absorbtance. Using a simple model of vegetation it was possible to study the effect of lateral discontinuity on the relationship between BHR/absorbtance/transmittance and (1) LAI and (2) solar zenith angle. The model was validated first, using RTE models (1D and 3D), second, using Monte Carlo simulations of a computer generated maize canopy, and finally, using field data from the Jornada PROVE field campaign. The general agreement is good. The major drawback of the model is approximate description of the hot spot effect. The numerical simulations of Mean RTE showed, that this equation as well as the classical stochastic equation does not describe this phenomenon, if one looks for solutions in class $C^2$ (standard continuously differentiable functions). Hence, we used an approximate model for the hot spot. Further model performance can be achieved by accurate modeling of geometrical shapes of vegetation canopy in order to utilize model simulations in biophysical parameter retrieval algorithms.

References


absorbed photosynthetically active radiation from atmosphere-corrected MISR data,


Stewart, R. (1990), Modeling radiant energy transfer in vegetation canopies, M.S. Thesis, Kansas State University, Manhattan, Kansas, 66506.


Figure 1: The coordinate system with vertical axis $z$ directed down. $H$ is height of canopy, $N$- direction to north, $\Omega(\theta, \varphi)$- is direction, with $\theta$ as zenith angle and $\varphi$ as azimuth angle.
Figure 2: Procedure for integration of RTE. Points A and B correspond to the starting points of integration (located on boundaries), which is performed along the direction $\theta$ up to the inner point C, which has the coordinates $(x, y, z)$. Points D and F designate any point located on lines AC and BC. The 3D equation of lines AC and BC is given (the parameter, controlling location on the line is $\xi$).
Figure 3: Comparison of BRF in the principal plane simulated by Mean RTE (solid line) and Monte Carlo methods (dashed line). In all cases soil is "dry", cab=50 and SZA=45 degrees.
Figure 4: Comparison of BRF in the principal plane simulated by Mean RTE (solid line) and Monte Carlo methods (dashed line). In all cases LAI=1.64, cab=50 and SZA=45 degrees. The "dry" soil case was used in order to estimate missing parameters and "wet" soil case was used for validation.
Figure 5: Comparison of DHR simulated by Mean RTE and Monte Carlo methods for "dry" and "wet" soil cases. 50 values of DHR were compared in the case of "dry" soil and 30 values for "wet" soil.
Figure 6: BHR, absorptance and transmittance as a function of LAI at red (left) and near-infrared (right) wavelengths evaluated with Mean RTE (solid line), TWOVEG (dashed line) and DISORD (doted line). In all cases SZA=30 degrees.
Figure 7: BHR, absorptance and transmittance as a function of solar zenith angle at red (left) and near-infrared (right) wavelengths evaluated with Mean RTE (solid line), TWOVEG (dashed line) and DISORD (doted line). In all cases LAI=5.
Figure 8: Comparison of BRF simulated by Mean RTE with Jornada PROVE field data at 1020 nm and 870 nm for 3 values of SZA: 20, 40 and 70 degrees.
Figure 9: Influence of clumping on the radiation regime in vegetation canopy at red (left) and near-infrared (right) wavelengths. BHR, absorptance and transmittance as a function of LAI evaluated at three values of ground cover, representing varying degree of clumping: P=1. (dashed line), P=0.75 (doted line), P=0.5 (solid line). In all cases SZA=30 deg.
Figure 10: Influence of clumping on the radiation regime in vegetation canopy at red (left) and near-infrared (right) wavelengths. BHR, absorptance and transmittance as a function of solar zenith angle evaluated at 3 values of ground cover, representing varying degree of clumping: P=1.0 (dashed line), P=0.75 (doted line), P=0.5 (solid line). In all cases LAI=5.
Table 1: Physical characteristics of 6 growth stages of maize canopy, Maria España et al. (in press)

<table>
<thead>
<tr>
<th>Phenological stage</th>
<th>Leaf stage</th>
<th>LAI</th>
<th>Plant height, [m]</th>
<th>Cover fraction</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>0.25</td>
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<td>24</td>
<td>6.25</td>
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<td>1.00</td>
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</table>

Table 2: Optical properties of the Maize leaves and the soil. Cab 50 designates chlorophyll a and b concentration of 0.5 g · m², Maria España et al. (in press).

<table>
<thead>
<tr>
<th>Wavelength, [nm]</th>
<th>430</th>
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<th>562</th>
<th>630</th>
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<th>740</th>
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<td>Leaf refl., Cab50</td>
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<td>Leaf trans., Cab50</td>
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<td>Dry soil hem. refl.</td>
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<td>0.1359</td>
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<td>Moist soil hem. refl.</td>
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