Atmospheric correction

Part 1. Ground-based measurements of BRDF and albedo

1. Atmospheric effect on albedo, dependence on surface type and solar zenith angle

Most completely, surface reflectance is described by the bidirectional reflectance distribution function (BRDF), which has been the subject of intensive theoretical and experimental research in the last two decades. This study has led to the creation of numerical databases and analytical models of the BRDF for a wide range of land covers.

Albedo is another characteristic of surface reflection, defined as a ratio of the reflected and incident fluxes on the surface level. As opposed to the BRDF, it depends on atmospheric conditions. The experimental and theoretical studies of the albedo’s dependence on atmospheric conditions or on solar zenith angle often give only qualitative or even contradictory results.

Questions to answer:
1. To what extent is the surface albedo an intrinsic property of a surface and how much is it affected by the atmospheric conditions?
2. How does it depend on the solar zenith angle for different types of land covers?

1.1 Analytical analysis

Lower boundary condition:

\[ I(\tau_0, \mu, \phi) = S_0 \mu_0 r(\mu_0, \mu, \phi - \phi_0) e^{-\tau_0/\mu_0} + \frac{1}{\pi} \int_0^{2\pi} d\phi' \int_0^1 r(\mu', \mu, \phi - \phi') I(\tau_0, \mu', \phi') \mu' d\mu' \]  

(1.1)

Let us separate a direct and diffuse components in the incident flux,

\[ F(\mu_0) = F_{\text{dir}} + F_{\text{dif}} \], where \( F_{\text{dir}} = \pi S_0 \mu_0 e^{-\tau_0/\mu_0} \) and \( F_{\text{dif}} = \int \mu' I(s') ds' \)

and in the reflected flux: \( F^\uparrow = \int_{\Omega^-} \mu I(s') ds' \). For the convenience of the following analysis, let us define the directional and diffuse components of the surface albedo:

\[ \rho = \rho_b F_{\text{dir}} + \rho_d F_{\text{dif}} \]  

(1.2)

where directional (\( \rho_b \)) and diffuse (\( \rho_d \)) albedo are defined as

\[ \rho_b(\mu_0) = \frac{1}{\pi} \int_{\Omega^-} \mu r(s_0, s) ds \]  

(1.2a)

\[ \rho_d = \frac{1}{\pi} \int_{\Omega^-} d\mu ds' \int_{\Omega^-} \mu' I(s') r(s', s) ds' / F_{\text{dif}} \]  

(1.2b)

where \( ds = d\mu d\phi \), and integration is performed over the upper (\( \Omega^+ \)) and lower (\( \Omega^- \)) hemispheres of directions. The above expressions show that the directional albedo is purely a surface property. It depends on solar zenith angle. Given the general bowl shape of BRDF, this component of albedo increases with \( \theta_0 \) following the increase of the BRDF. Diffuse albedo depends both on atmospheric and on surface parameters, and it is angular independent. In the total albedo, these components are weighted with the relative direct or diffuse incident fluxes.
Let us perform a heuristic analysis of the directional and diffuse albedo, which are the
extreme cases of $\rho(\mu_0)$ at small ($\tau_0 \rightarrow 0$) and large optical thickness ($\tau_0 \rightarrow \infty$), respectively. The
analysis can be done analytically for the BRDF model of Minnaert [1941]

$$ r(\mu', \mu) = \frac{k_m + 1}{2} \cdot q(\mu') \cdot \mu^{-1} $$  \hspace{1cm} (1.3) 

Parameter $k_m$ of this model defines the steepness of BRDF, or its anisotropy, and the
magnitude of BRDF and albedo depends on parameter $q$. Despite its simplicity, this model
adequately describes the general bowl shape of BRDF, and it will allow us to obtain
qualitatively correct results.

At $\tau_0 \rightarrow 0$, there is only direct unattenuated radiation, and the directional albedo is simply

$$ \rho_b = q \mu_0^{k_m -1} $$  \hspace{1cm} (1.4) 

At $\tau_0 \rightarrow \infty$, all the incident radiation is diffuse and “saturated” in a sense that its angular
distribution changes little with the further increase of $\tau_0$. It does not depend on azimuth. The
intensity of radiance gradually decreases from zenith to horizon (Figure 2), which can be verified with the unaided eye in conditions of uniformly overcast cloudy sky. In practice, the
saturation takes place starting from $\tau_0 = 10 - 20$. To a good accuracy, the incident radiance is a
linear function of a cosine of view angle

$$ I(\tau_0, \mu') = a + \mu' b $$  \hspace{1cm} (1.5) 

Using relation (5), it is easy to obtain the expression for the diffuse albedo

$$ \rho_d = q \frac{k_m + 1}{2} + \frac{k_m + 2}{b/a} $$  \hspace{1cm} (1.6) 

To calculate $\rho_b$ and $\rho_d$ numerically, we need to estimate the ratio $b/a$ and the range of
variation of parameter $k_m$ for natural surfaces. From Figure 2, one can find out that despite
that parameters $a$ and $b$ depend on solar zenith angle and cloud optical depth, their ratio $b/a$ is
nearly constant and equal to $\approx 1.7 - 1.8$. Also, from larger sets of analyzed and published BRDF
data fitted by analytical models, it follows that typical values of factor $k$ are confined in the
range $0.5 - 0.8$. Numerical estimates for the normalized quantities $\rho_b/q$ and $\rho_d/q$ based on the
above assessments are compiled in Table 1.

**Table 1.** Normalized Directional ($\rho_b/q$) and Diffuse ($\rho_d/q$) Albedo Calculated for the BRDF
Model of Minnaert at Different Values of the Surface Anisotropy Parameter $k_m$

<table>
<thead>
<tr>
<th>$k_m$</th>
<th>$\rho_d/q$, $\theta_0 = 10^\circ$</th>
<th>$\rho_d/q$, $\theta_0 = 45^\circ$</th>
<th>$\rho_d/q$, $\theta_0 = 51^\circ$</th>
<th>$\rho_d/q$, $\theta_0 = 70^\circ$</th>
</tr>
</thead>
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<tr>
<td>0.8</td>
<td>1.09</td>
<td>1.03</td>
<td>1.07</td>
<td>1.09</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.072</td>
<td>1.097</td>
</tr>
<tr>
<td>0.5</td>
<td>1.26</td>
<td>1.008</td>
<td>1.189</td>
<td>1.261</td>
</tr>
<tr>
<td></td>
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<td>1.261</td>
<td>1.710</td>
</tr>
<tr>
<td>0</td>
<td>1.73</td>
<td>1.015</td>
<td>1.414</td>
<td>1.589</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.589</td>
<td>2.924</td>
</tr>
</tbody>
</table>

The directional albedo is given for several different solar zenith angles.

From data of Table 1, one can make several observations assuming monotonic
dependence of albedo on optical thickness. First, at small solar zenith angles $\rho_b(\theta_0) < \rho_d$, so
the surface albedo should increase as $\tau_0$ increases. At big angles $\theta_0$ the dependence is inverse
since $\rho_b(\theta_0) > \rho_d$, and surface albedo should decrease as $\tau_0$ increases. Second, when the solar
zenith angle is about $51^\circ$, the diffuse albedo is approximately equal to the directional albedo
\( \rho_s \equiv \rho_b \left( 51^\circ \right) \). Therefore surface albedo at this particular angle of sun will not be sensitive to the atmospheric opacity. Third, the range of albedo variation with \( \tau_0 \), naturally, correlates with the degree of anisotropy of surface reflectance. These conclusions are verified with the results of our numerical research (Figures).

- In a certain range of solar zenith angles (52°-57°) the atmospheric influence on albedo is minimal. This effect exists for all land covers we have studied, and probably, it has a universal nature. Albedo measurements at these solar angles may be used for the surface characterization without correction for atmospheric variability.

- The variation of albedo with \( \tau \) is largest at large solar angles, and given the surface anisotropy, the atmospheric effect will be mostly significant over the polar regions. Somewhat smaller variation can be expected at a high Sun, for example, in the equatorial zone, and minimal variation should be observed in midlatitudes.

- Variability of albedo with \( \tau \) depends on the type of surface, more precisely, on anisotropy of its reflectance. On a qualitative level, anisotropy is a steepness of the BRDF. For the Minnaert model, it can be defined as \( |k_m - 1| \), \( k_m = 1 \) for a Lambertian surface. The atmospheric effect on albedo increases as \( |k_m - 1| \) increases.

A separate and very interesting question is the dependence \( \rho(\theta_0) \) at large solar angles. Even at the clear atmosphere, the exponential attenuation of the direct radiation with the increase of the optical path \( (\tau_0/\cos\theta_0) \) as \( \theta_0 \) grows will eventually prevail over the increase of BRDF. Therefore starting from some \( \theta_0 \), the surface albedo should drop off (Figure) [Kondratyev, 1969, p. 439].

Summary:

1. For typical surfaces the maximal range of surface albedo variation with atmospheric opacity is relatively small. It does not exceed 10-15% of total albedo value at \( 0 \leq \theta_0 \leq 50^\circ \) and 20-30% at \( 0 \leq \theta_0 \leq 70^\circ \). Within reasonable limits of aerosol optical depth, i.e. \( 0.05 \leq \tau \leq 0.5 \), and of sun zenith angle, \( 30^\circ \leq \theta_0 \leq 63^\circ \), the range of variation narrows down to 6-8%. In this sense, within these ranges, albedo is primarily an intrinsic property of surface and its dependence on atmospheric conditions has a relatively minor effect. Nevertheless, the atmosphere-induced variability of albedo may exceed the currently accepted tolerance of climatic models, especially over vegetation in the near-IR, so albedo calculations/measurements should take the atmospheric effect into account.

2. In the clear-sky conditions, aerosol optical thickness is the main atmospheric factor affecting albedo. The variability of scattering function and of single-scattering albedo is much less important, the typical effect is less than 1% of the albedo value, and can be neglected.

3. For small solar zenith angles, surface albedo increases with atmospheric opacity, while for big angles \( \theta_0 \), it decreases. The watershed lies at \( \theta_0 = 52^\circ - 57^\circ \), where it is almost insensitive to the atmospheric opacity. Albedo measurements at these angles can be used directly for the surface characterization without accounting for atmospheric variability.

4. Given the surface anisotropy, one should expect larger variations of the surface albedo with atmospheric opacity over the polar regions and on the equator and smaller variations at the middle latitudes.

5. At large solar zenith angles (\( \theta_0 > 70^\circ - 80^\circ \)), albedo drops off with a further increase of the angle.

2. BRDF retrieval from “BRDF measurements”
Experimental study of the directional reflective properties of vegetated land covers in the visible and near-IR parts of the spectrum was initiated by Kriebel [1974] in the early 1970s. A major problem in deriving the BRDF from measurements is accounting for the diffuse skylight. Deering and Eck [1987] proved experimentally that the surface reflected radiance strongly depends on turbidity of the atmosphere.

2.1 Accuracy of BRDF Measurements

Most experimental BRDF reports fall into three major groups: (1) an apparent reflectance, $R_1 = I / S \lambda \mu_0$, [Eaton and Dirnhirn, 1979; Deering and Eck, 1987]; (2) reflectance normalized for the direct illumination,

$$R_2 = \frac{e^{\tau_0/\mu_0}}{\mu_0} I(s_0, s) / S \lambda \mu_0$$

and (3) reflectance, obtained with a reference panel

$$R_3 = \pi I(s_0, s) / F(\mu_0)$$

[e.g., Kimes et al., 1986; Kimes and Newcomb, 1987]. Measurements of the third group are most accurate, however none of these groups gives the true BRDF. Thus, a significant amount of data in the currently accumulated databases of the spectral-directional surface reflectance have only approximate or no correction for atmospheric effects. Moreover, until now the magnitude of atmospheric distortions in the reported BRDF has not been clear.

To fill this gap, we conducted an error study for the reflectance factors $R_1 - R_3$ at three wavelengths, 0.45, 0.65 and 0.85 µm. Figures 5a and 5b show the relative errors ($r/R-1$, %) in the principle plane for grass in the visible and near-IR ranges. One can see that the BRDF is always underestimated in group 1 and overestimated in group 2. Group 2 gives the largest error proportional to the relative diffuse incident flux. Group 3 gives the “centered” reflectance factor, so that after being integrated over angles, it should predict the albedo, which is accurate to the factor of multiple reflections between the surface and the atmosphere. However, even in this group at $\tau_0=0.1$ the shape of BRDF can be considerably distorted, up to ±12% in the red and ±8% in the near-IR.

Figures 6a and 6b show the relative errors in the corresponding MRPV parameters and surface albedo in groups 2 and 3. The errors are considerable in group 2 when albedo for the aerosol-free atmosphere is overestimated by up to 19% at 0.45 µm, by 5% at 0.65 µm, and by 2.4% at 0.85µm. At $\tau_0=0.1$, these errors increase up to 39%, 18%, and 19%, respectively.

In group 3, deviation of albedo from the accurate value is due to the multiple reflection of light between the surface and the atmosphere. The effect is stronger over a brighter surface.

Over surfaces of medium and high reflectance, the errors shown may considerably exceed the tolerance of climatologic models that require an accuracy of ±0.02 in the global albedo data sets [Sellers, 1993].

The first iterative methods for accurate biconical reflectance/BRDF retrieval were developed by Kriebel [1974] and Martonchik [1994] based on radiative transfer solutions. Both methods require measurements to be performed for a representative number of solar zenith angles and use interpolation to obtain missing data between the view and incident angles of measurements.

2.2 The New Algorithm

The lower boundary condition of the radiative transfer equation allows one to find the reflected radiance $I(\tau_0, s)$ in the direction $\nu$ for the specified surface BRDF $r(\nu', \nu')$.
\[ I(\tau_0, s) = S_\lambda \mu_0 r(s_0, s)e^{-\frac{\tau_0}{\mu_0}} + \frac{1}{\pi} \int_0^{2\pi} d\phi' \int_0^1 r(s', s)I(\tau_0, s') \mu' d\mu' \]  \hspace{1cm} (2.1)

Here \( S_\lambda \) is the extraterrestrial solar irradiance; \( \tau_0 \) is the atmospheric optical thickness. The incidence and reflectance directions \( s', s \) are defined by zenith and azimuthal angles \( (\theta, \varphi) \), \( \mu = \cos \theta \) and \( \mu_0 = \cos \theta_0 \).

Let us assume that both the reflected and incident radiance fields are measured on some grid of angles \( s_i = (\mu_i, \varphi_i) \), \( s'_i = (\mu'_n, \varphi'_m) \), \( i, n = 1, \ldots, N \), \( j, m = 1, \ldots, M \) with a sensor of small aperture that will let us deal with the bidirectional rather than with the biconical reflectance. Equation (1) provides the basis for determining the pointwise function \( r(s'_i, s_i) \). If the measurements of the sky and reflected radiance are performed for a complete set of solar zenith angles \( \{\mu_0\} \), which includes all zenith view angles \( \{\mu_0_n\} \subset \{\mu'_n\} \), the discussed problem becomes the well known Fredholm equation of the 2nd kind with a known kernel \( I(\tau_0, s') \). Under clear-sky conditions, it has a unique solution, which can be found with the resolvent method [Mikhlin, 1957].

At large optical thickness, the first term of (2.1) becomes negligibly small, and (2.1) turns into a Fredholm equation of the 1st kind. The inverse problem in this case is ill posed and requires a special regularization in order to confine the multitude of plausible solutions.

We will consider only the experimentally significant case of medium to high transparency of the atmosphere. Owing to experimental constraints in field conditions or atmospheric instability, reliable measurements are frequently available only for a single or a small number of solar zenith angles \( \{\mu_0\} \). In this case there are insufficient data to use the standard way of solving the Fredholm equation. However, we can use a priori information on the general BRDF shape in a form of analytical model, whose parameters can be found from measurements made at single solar angle. The analytical model is required to extend the estimate of BRDF at a given solar angle \( \theta_0 \) to the upper hemisphere of directions in order to evaluate the integral term of equation (2.1). The resulting BRDF in our algorithm is a composite function having accurate values in the directions of measurements, corresponding to measured intensities, and model values at other directions.

We require the analytical model to have properties of rotational symmetry and reciprocity. Rotational symmetry implies that the BRDF depends only on relative azimuth. As mentioned by Kriebel et al. [1996], this property holds as long as the surface has no strong linear structure.

The main mechanism behind the reciprocity violation is lateral inhomogeneity of the observed and surrounding area. First, it causes a nonzero horizontal photon transport from brighter to darker area, which breaks the reciprocity [Girolamo et al., 1998]. Second, the averaging area, and therefore the degree of nonuniformity of the observed scene, depends on the view zenith angle. This may violate both reciprocity and rotational symmetry. Therefore we assume further that the BRDF measurements are performed in the far field and the pixel is large enough [Snyder, 1998]. Practically, this means that the observed area at all view angles should be much larger than the size of primary structure elements of the surface.

Let \( I(s_0, s_i) \) and \( I^{(n)}_i(s_0, s_i) \) denote the measured radiance in the direction \( s_i \) and the theoretical radiance at \( n \)th iteration, respectively. To solve the problem, we use the following iterative algorithm:

(1) calculate
\[ r_i^{(n)} = r_i^{(n-1)} + \alpha \Delta r_i^{(n)} \] \hspace{1cm} (2.2)

where
and $\alpha$ is a relaxation parameter (weight factor) which accelerates convergence.

2. Find the best fit parameters of the analytical BRDF model for the pointwise function $r_{t_i}^{(n)}$.

3. Compute the theoretical radiance $I_t^{(n)}(s_0, s_i)$ corresponding to the composite BRDF, which is equal to $r_{t_i}^{(n)}$ at angles $s_i$ and to the values of analytical model at other directions.

4. Evaluate the standard deviation $\sigma = \sqrt{\sum (I(s_0, s_i) - I_t^{(n)}(s_0, s_i))^2}$.

5. Repeat steps 1-4 until $\sigma \leq \epsilon$, where $\epsilon$ is a given error threshold.

The use of the composite function allows us to retain specific features of measured reflectance in the directions of measurements, which may be useful in the validation of BRDF models and retrieval of surface physical properties. Parameters of the analytical BRDF model potentially have a number of applications, from realistic radiative transfer simulations to land cover classification. Note that the choice of the analytical model has no significant effect on the retrieval as long as the model fits well the general shape of the measured BRDF.

To solve the best fit problem in step 2, we use the modified RPV (MRPV) function, where the Heney-Greenstein term is substituted by an exponential function $\exp(b \cos \gamma)$, where $\gamma$ is a scattering angle. This modification of Martonchik et al. [1998] allows one to linearize the MRPV function for an efficient inversion. We use the following algorithm: logarithmic transformation of the MRPV function $R$ linearizes it for the unknowns $k$ and $b$

$$\ln R = (k-1) \ln \mu_0 (\mu + \mu_0) + b \ln \cos \gamma + \ln \rho (1 + \frac{1 - \rho}{1 + G})$$

(2.4)

Parameters $k$, $b$, $\rho$ are then found from the least squares problem,

$$F = \sum r_i^2 (\ln R_i - \ln r_i)^2 = \min \{k, b, \rho\}$$

(2.5)

which is expanded into three explicit equations following from

$$\frac{\partial F}{\partial k} = 0, \quad \frac{\partial F}{\partial b} = 0, \quad \frac{\partial F}{\partial \rho} = 0$$

(2.6)

The first two equations, linear in parameters $k$ and $b$, can be solved with respect to these two parameters. The corresponding expressions are substituted into the third equation, which gives the nonlinear equation for the unknown $\rho$, $f(\rho) = 0$. The function $f(\rho)$ is continuous and takes opposite signs on the borders of interval $(0, \rho)$, which confines the range of parameter $\rho$. The value $\rho$ is an arbitrary large number, say, $\rho = 100$. On the basis of these properties, we use the Wijngaarden-Dekker-Brent algorithm [Press et al., 1992] to find a root of $f(\rho) = 0$.

Having found $\rho$, it is easy to find parameters $k$ and $b$ analytically. Compared to nonlinear minimization, this approach is much faster and less demanding of computational resources.

The form of the merit function, $F = \sum r_i^2 (\ln R_i - \ln r_i)^2$, provides an unbiased solution equivalent to the result of minimization of the standard form $F_{st} = \sum (R_i - r_i)^2$. To prove that, it is sufficient to show that $F \rightarrow F_{st}$ in the vicinity of the minimum. Near the minimum, $R_i/r_i = 1 + \delta_i$, where $\delta_i < 1$ for all $i$, provided that the analytical function $R_i$ approximates the measurements uniformly. Therefore one can expand the logarithm into a Taylor series retaining only a linear term,

$$r_i (\ln R_i - \ln r_i) = r_i \ln \frac{R_i}{r_i} \rightarrow r_i \delta_i = R_i - r_i$$

(2.7)

which leads us to the standard form $F_{st}$. 

Another form of merit function, \( \sum (\ln R_i - \ln r_i)^2 \), is sensitive to noise in the data, which may bias the solution. In this case, the random errors of measurements \( \varepsilon_i \) are amplified in the vicinity of the minimum as \( \varepsilon_i / r_i \) since \( r_i < 1 \) or even \( r_i << 1 \) in the meaningful domain of view angles, except for relatively large ones where \( r_i \geq 1 \).

### 2.3 Properties of Numerical Scheme

At small and medium atmospheric opacity (\( \tau \leq 0.7 \)), the algorithm’s convergence is stable and fast. It is not sensitive to the initial estimate of BRDF, which becomes an issue at larger optical thickness. The initial estimate can be obtained either from the direct (2.8a) or from the total surface irradiance (2.8b)

\[
\begin{align*}
    r^{(0)}(s_0, s_i) &= e^{\tau_i/\mu_0} I(s_0, s_i) / S \mu_0, \quad \text{(constraint } F_{\text{dir}} / F \geq 0.1 - 0.15) \quad (2.8a) \\
    r^{(0)}(s_0, s_i) &= \pi I(s_0, s) / F(\mu_0), \quad \text{(no constraints)} \quad (2.8b)
\end{align*}
\]

where \( F(\mu_0) \) is the total downward flux.

**Number of Iterations to Convergence and Weight Factor (Fig.)**

Depending on optical thickness, the weight factor \( \alpha \) can be selected optimally to maximize convergence. E.g., at \( \tau < 0.7 \) and optimal weight, this algorithm is very efficient and requires only two to six iterations.

### 2.4 Conclusions

We have described a new algorithm for the accurate retrieval of surface BRDF from ground-based measurements. Compared to methods of Kriebel [1974] or Martonchik [1994], which need measurements at a representative number of solar zenith angles, this method is able to obtain the BRDF from measurements at a single solar angle. This improvement allows one to process field data despite an incomplete set of measurements, as may be the case on partly cloudy days. On the other hand, our method works similarly to the method of Martonchik if measurements are available for many solar angles.

The accuracy of BRDF/albedo retrieval is limited by our approximate knowledge of aerosol parameters. At maximal uncertainty, the relative error in surface albedo determined by the retrieved BRDF does not exceed \( \pm 3-4\% \) at \( \tau^a \leq 0.5 \). The errors of MRPV parameters \( \rho \) and \( b \) can be a factor of 2-3 higher. At this point, however, we cannot make a good judgement about the sufficiency of this accuracy. The answer to this question should come from practical applications such as land cover classification or the derivation of physical surface properties.

One of the purposes of this work was to estimate errors in the uncorrected BRDF data usually reported in the literature. Our simulations have shown that the distortions of the BRDF shape are considerable even at low aerosol optical thickness. For example, the reference panel technique measures the BRDF with an error of up to \( \pm 12\% \) in the red and \( \pm 8\% \) in the near-IR at \( \tau^a = 0.1 \). These results suggest that the atmospheric correction should be a required step in the processing of field reflectance data.
Atmospheric correction

Part 2. Generic AC algorithm for satellite measurements

Atmospheric correction is designed to retrieve the true 2D surface reflectance from satellite measurements at the TOA. The algorithm should deal with the following effects:
- atmospheric (molecular and aerosol) scattering and absorption of solar radiation;
- 3D radiation effects (finite size of pixel and adjacency effect);
- bi-directional properties of surface reflectance.

1. 3D effects, Lambertian surface

The retrieval of 2D field of surface reflectance requires application of 3-D radiative transfer theory. However, traditional analysis of satellite data is based on a classical 1-D theory \cite{Chandrasekhar, 1960}, which assumes the surface to be infinite and uniform. Non-homogeneity of surface gives rise to the horizontal fluxes of radiation in the atmosphere, directed from the brighter surface areas to the darker areas. As a result, after several instances of scattering in the atmosphere, some of the photons reflected from the bright surface may be detected over the dark pixels (adjacency effect) \cite{Otterman and Fraser, 1979}. In satellite images, these 3-D radiation effects appear as blurring of the fine spatial structure of the surface and reduction of the image contrasts \cite{Mekler and Kaufman, 1980; Pearce, 1977}. The magnitude of these distortions depends on a number of factors such as the contrast and scale of horizontal variations of surface reflectance, spatial imagery resolution, viewing geometry, and atmospheric conditions, with the aerosol optical thickness and effective height of scattering being the major factors \cite{Kaufman, 1982; Diner and Martonchik, 1985}.

It is important to note that the adjacency effect has a systematic nature in that it always increases apparent reflectance of dark surface targets compared to their true reflectance, and decreases apparent reflectance of bright objects.

The following section describes a theoretical treatment of 3-D radiative transfer problem \cite{Sushkevich et al., 1990}. It allows one to obtain a boundary-value problem for OTF by linearizing a 3-D problem and to develop a powerful and flexible numerical method for the direct and inverse calculations over an arbitrarily non-uniform Lambertian surface.

1.1 Theoretical Approach

Let us consider a horizontally homogeneous atmosphere bounded by a Lambertian non-uniform surface with albedo $\rho(r, \phi = \alpha(x, y))$. Given the atmospheric scattering $\sigma(z)$ and extinction coefficients $\alpha(z)$ and scattering function $\chi(z, \gamma)$, the radiance at the altitude $z$ in the direction $s=(\mu, \phi)$, $\mu = \cos \theta$, can be found as a solution of 3-D boundary-value problem,

\[
(\nabla \cdot \mathbf{I}(z; r; s)) + \sigma(z) \mathbf{I}(z; r; s) = \frac{\sigma(z)}{4\pi} \int_{\Omega} \chi(z, \gamma) \mathbf{I}(z; r; s') ds' + \frac{1}{4} \sigma(z) \chi(z, \gamma_0) S_\lambda e^{-\gamma_0} \quad (1a)
\]

\[
I(0; r; s) = 0, \mu > 0; \quad (1b)
\]

\[
I(H; r; s) = \rho(r) \{ S_\lambda \mu_0 e^{-\gamma_0} + \frac{1}{\pi} \int_{0}^{2\pi} d\phi \int_{0}^{1} I(H; r; s') \mu d\mu' \}, \mu < 0 \quad (1c)
\]
where $\tilde{\varphi} = (\sqrt{1 - \mu^2} \cos \varphi, \sqrt{1 - \mu^2} \sin \varphi, \mu)$ and $\tilde{\varphi} = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})$. $S_\lambda$ is the extraterrestrial solar irradiance at wavelength $\lambda$, and scattering function is normalized by $4\pi$. Z-axis is pointed downward, so cosine of zenith view angle $\mu>0$ for downward directions, and $\mu\leq0$ otherwise.

Separation of the mean and variation in surface albedo and radiance

$$\rho(r) = \bar{\rho} + \tilde{\rho}(r), \quad I(z; r; s) = \bar{I}(z; s) + \tilde{I}(z; r; s)$$  

allows one to divide general problem (1) into two sub-problems. One of them is a classical 1-D problem for the mean radiance with uniform boundary condition corresponding to the mean albedo. The other one is the problem for the radiance variation:

$$\mu \frac{\partial \tilde{I}}{\partial z} + \sqrt{1 - \mu^2} \{ \cos \varphi \frac{\partial \tilde{I}}{\partial x} + \sin \varphi \frac{\partial \tilde{I}}{\partial y} \} + \alpha(z) \tilde{I} = \frac{\sigma(z)}{4\pi} \int_{\Omega} \chi(z, \gamma) \tilde{I}(z; r; s') ds'$$  

(3a)

$$\tilde{I}(0; r; s) = 0, \mu>0;$$  

(3b)

$$\tilde{I}(H; r; s) = S_\lambda \mu_0 e^{-\mu_0} \tilde{\rho}(r) + \frac{1}{\pi} \int_{\Omega} (\tilde{I}(H; r; s') \tilde{\rho} + \tilde{I}(H; s') \tilde{\rho}(r) + \tilde{I}(H; r; s') \tilde{\rho}(r)) \mu' ds' \quad \mu<0.$$

(3c)

This problem is non-linear in surface albedo variation due to multiple reflections of photons between the atmosphere and surface albedo variations. These interactions are described by the last term of the lower boundary condition (3c). The non-linear contribution is small and bounded from above [Sushkevich et al., 1990] as

$$\tilde{I}_{nl}(z; r; s) / \tilde{I}(z; r; s) \leq \frac{c_0 \rho_{\text{max}}}{1 - c_0 \bar{\rho}},$$

where $c_0$ is spherical albedo of atmosphere, and $\rho_{\text{max}}$ is maximal albedo value in image. In clear-sky conditions, $\tilde{I}_{nl}(z; r; s)$ does not exceed several percents of the variation of radiance and is even smaller with respect to the total signal. This fact allows us to neglect this term in (3c) and linearize problem (3). Finally, applying Fourier-transform $F[\tilde{\rho}(r)] = \tilde{\rho}(p)$, $F[\tilde{I}(z; r; s)] = \tilde{I}(z; p; s)$, $p = (p_x, p_y)$, and introducing an optical transfer function $\psi_1(z; p; s)$ via a linear model

$$\tilde{I}(z; p; s) = \tilde{\rho}(p) \psi_1(z; p; s) E(\mu_0)$$

(4)

with $E(\mu_0) = S_\lambda \mu_0 e^{-\mu_0} + \frac{1}{\pi} \int_{\Omega} \tilde{I}(H; s') \mu' ds'$ being the mean surface illuminosity, we arrive at the problem for atmospheric OTF:

$$\mu \frac{\partial \psi_1}{\partial \tau} + [1 - i \sqrt{1 - \mu^2} \{ p_x \cos \varphi + p_y \sin \varphi \} / \alpha(z)] \psi(\tau, p, s) = \frac{\alpha(\tau)}{4\pi} \int_{\Omega} \chi(\tau, \gamma) \psi(\tau; p; s') ds',$$

(5a)

$$\psi(0; p; s) = 0, \mu>0; \quad \psi(\tau_0; p; s) = 1, \mu<0.$$

(5b)

Eqs. (5a)-(5b) represent a set of parametric problems to be solved at different values of vector-parameter $p$. Thus introduced OTF does not depend on surface properties. As such, it is very convenient for studies of the influence of different atmospheric parameters as well as for 3-D radiative transfer calculations. Function $\psi_1(z; p; s)$ introduced by formula (4) is linearly related to the solution of problem (5) $\psi(z; p; s)$ and depends on the mean surface albedo as...
\[ \psi_1(z; p; s) = \psi(z; p; s) / [1 - \bar{c}(p)], \quad c(p) = \frac{1}{\pi} \int_0^\pi \psi(H; p; s') \mu' ds' \]  

(6)

c(p) is spherical albedo of atmosphere at spatial frequency \( p \) defined by analogy with the corresponding value \( c_0 \equiv c(0) \) of 1D theory.

Below, we consider the case of one-dimensional surface where \( p \equiv p_x, p_y = 0 \). In the upward directions, OTF is expressed as a sum of the direct and diffuse transmittance

\[ \psi(\tau; p; s) = e^{-\xi_{0,\tau}/|\mu|} + A(\tau; p; s)e^{i\Phi(\tau, p, s)}, \]  

(7)

where \( x_1 = (H-z)\tan\theta \cos\phi \) is a geometrical shift in the coordinate for slant observations. Note that the diffuse OTF is a complex function, given by its amplitude \( A \) and phase \( \Phi \). Based on given definitions (2), (4), (6), (7), the radiance at the top of atmosphere is represented as a sum of three terms: mean radiance, corresponding to the mean albedo, and direct and diffuse components of radiance variation

\[ I(x; s) = \bar{I}(s) + E(\mu_0) \{ \bar{\rho}(x - x_1) e^{-\xi_{0,\tau}/|\mu|} + \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{\rho}(p) A(p, s) e^{-i\int(p - x, \tau)} \Phi(p, s) dp \} \]  

(8)

\[ \bar{I}(s) = D + \bar{\rho} E(\mu_0) T(\mu), \quad E(\mu_0) = \frac{E_0(\mu_0)}{1 - \bar{\rho} c_0} \]  

(9)

In formula (9), \( E_0(\mu_0) \) is surface illuminosity created by the direct sun light and path radiance \( D \). Described formalism is accurate to within the non-linear term, which was neglected in the lower boundary condition (3c). However, the described approach allows to calculate the non-linear radiance using linear OTF \( \psi(\tau; p; s) \). Consideration of successive orders of reflection from surface albedo variation in problem (3) leads to the following expression [Sushkevich et al., 1990]

\[ I^n(x; s) = E(\mu_0) \sum_{k=1}^\infty \frac{1}{(2\pi)^k} \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} \bar{\rho}(p - p_1)\bar{\rho}(p_1 - p_2)\ldots \bar{\rho}(p_{k-1}) \psi_k e^{-i\int(x - x_1) dp_{k-1} \ldots dp} \]  

(10)

where index \( k \) represents different orders of reflection from albedo variation, and \( \psi_k \) is the optical transfer function of \( k \)th order. In case of Lambertian surface, function \( \psi_k \) is related to the linear OTF as follows:

\[ \psi_k(p, p_1, \ldots p_{k-1}, s) = \frac{\psi(p, s)}{1 - \bar{\rho} c(p)} U(p_1) \ldots U(p_{k-1}), \quad \text{where} \quad U(p_i) = \frac{c(p_i)}{1 - \bar{\rho} c(p_i)}. \]

The described analytical approach allows one to calculate functions \( \psi(z; p; s) \) and \( c(p) \) once for specified atmospheric conditions, and then use them in radiance calculations for arbitrary realizations of surface albedo. At present, we have obtained a solution of problem (5) at \( p_y = 0 \) (1-D OTF) using an effective SVD-modification of the spherical harmonics method with the smoothing procedure of integration of the source function [Lyapustin and Muldashev, 2000]. Our algorithm obtains a full multiple scattering solution in stratified atmosphere of arbitrary optical thickness with an accuracy of about 1%.

In addition to formula (8), let us give another form of expression for radiance via point-spread function, which is widely used in the theory of linear systems. Defining PSF as an inverse Fourier-transform of the OTF,
\[ PSF_p(x,s) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{A(p,s)}{1 - \tilde{p}(p)} e^{-i[px - \Phi(p,s)]} dp \]  

we can write:

\[ I(x;s) = \tilde{I}(s) + E(\mu_0) \{ \tilde{\rho}(x-x_1) e^{\tilde{\tau}_0/|\mu|} + \int_{-\infty}^{\infty} \tilde{\rho}(x'-x_1)PSF_p(x-x_1-x',s)dx' \} \]  

1.2 Properties of OTF

- Contrary to the direct transmission, the diffuse OTF increases with zenith view angle, thus enhancing adjacency effect.
- The range of essentially non-zero OTF-values narrows down with the increase of both zenith view angle and height of the atmosphere which via the similarity theorem reflects the broadening of PSF. The amplitude of OTF becomes negligibly small for some \( p > \bar{p} \), meaning that the surface objects smaller than certain size \( \Delta L < \bar{L} \) will be seen only in the signal directly transmitted through the atmosphere but not in the variation of the diffuse radiance. For reference, let us mention that the value \( p = 10 \text{ cycles/km} \) approximately corresponds to \( \Delta L = 300 \text{ m} \).
- The cut-off frequency \( \bar{p} \) decreases with zenith view angle and with the height of atmospheric layer. It means that the diffuse radiance variation becomes less and less sensitive to the small-scale surface objects as angle \( \theta \) or height \( H \) increase.
- The descending character of curves \( c(p) \) shows that the atmospheric backscattering at higher frequencies effectively decreases. This, in turn, leads to a fast drop of the relative contribution of the non-linear component of radiance variation (Eq. 10) with increasing spatial frequency.

Phase of OTF does not have direct physical interpretation. Analysis of problem (5) allows to establish the following symmetry properties of the amplitude and phase [Sushkevich et al., 1990]

\[ A(\pi - \varphi) = A(\varphi), \quad \Phi(\pi - \varphi) = -\Phi(\varphi) \]

1.3 Example: AC algorithm for Landsat-type data

- Lambertian case
  a) Average the radiance and find mean albedo: \( \bar{\rho} = \frac{\tilde{I} - D}{E_0 T + (\tilde{I} - D)c_0} \)
  b) Compute direct FFT for radiance variation \( F[\tilde{I}(z;\tilde{r};s)] = \tilde{I}(z;\tilde{p};s) \)
  c) Compute F-transform of albedo variation \( \tilde{\rho}(\tilde{p}) = \tilde{I}(z;\tilde{p};s)/[E(\mu_0)\psi(\tau;\tilde{p};s)] \)
  d) Compute inverse FFT: \( \tilde{\rho}(\tilde{r}) = F^{-1}[\tilde{\rho}(\tilde{p})] \)
  e) Compute surface albedo: \( \rho(\tilde{r}) = \bar{\rho} + \tilde{\rho}(\tilde{r}) \)

- Non-Lambertian case
  a)–b) and Determine dominant land cover type
  c) Compute F-transform of albedo variation \( \tilde{\rho}(\tilde{p}) = \tilde{I}(z;\tilde{p};s)/\psi_{BRDF}(\tau;\tilde{p};s) \)
  e) Correct the mean albedo for the BRDF effect (offset depends on land cover, solar zenith angle and \( \tau \)) to obtain nadir reflectance
2. The general AC algorithm

2.1 Boundary-value problem

Denote: \( r = (x, y); s = (\mu, \phi); \rho(r; s' \rightarrow s) \) is spatially dependent BRDF.

\[
\tilde{\mathbf{\nabla}} I(z; r; s) + \alpha(z) I(z; r; s) = \frac{\sigma(z)}{4\pi} \int_{\Omega} \chi(z, \gamma) I(z; \tau; \gamma') ds' + \frac{1}{4} \sigma(z) \chi(\tau, \gamma) S_0 \ e^{-\frac{\gamma}{\tau}}
\]

(1)

\( I(0; r; s) = 0; \)

(1a)

\( I(H; r; s) = S_0 \mu_0 e^{-\frac{\gamma_0}{\mu_0}} \rho(r; s_0 \rightarrow s) + \frac{1}{\pi} \int I(H; r; \gamma') \rho(r; s' \rightarrow s) \mu' ds' \)

(1b)

where \( \tilde{s} = (\sqrt{1 - \mu^2} \cos \phi, \sqrt{1 - \mu^2} \sin \phi, \mu), \ \tilde{\mathbf{\nabla}} = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right). \)

Let us separate the average and spatial variation in BRDF and radiance:

\[
\rho(r; s' \rightarrow s) = \bar{\rho}(s' \rightarrow s) + \tilde{\rho}(r; s' \rightarrow s), I(z; r; s) = \bar{I}(z; s) + \tilde{I}(z; r; s)
\]

Then the problem for variation is:

\[
(\tilde{\mathbf{\nabla}} \bar{I})(z; r; s) + \alpha(z) \bar{I}(z; r; s) = \frac{\sigma(z)}{4\pi} \int_{\Omega} \chi(z, \gamma) \bar{I}(z; \tau; \gamma') ds'
\]

(2)

\( \bar{I}(0; r; s) = 0; \)

(2a)

\( \tilde{I}(H; r; s) = S_0 \mu_0 e^{-\frac{\gamma_0}{\mu_0}} \bar{\rho}(r; s_0 \rightarrow s) + \frac{1}{\pi} \int (\tilde{I}(H; r; \gamma') \bar{\rho}(r; s' \rightarrow s) + \tilde{I}(H; r; \gamma') \bar{\rho}(r; s' \rightarrow s)) \mu' ds' \)

(2b)

Let us neglect the last term in (2b) in order to linearize problem (2). Next, factorize the surface reflectance:

\[
\bar{\rho}(s' \rightarrow s) = \bar{\rho} R(s' \rightarrow s); \bar{\rho}(r; s' \rightarrow s) = \bar{\rho}(r) \tilde{R}(s' \rightarrow s),
\]

where \( R(s' \rightarrow s) \) is a spatially average normalized BRDF. In order to solve problem (2), let us assume that the shape of BRDF is the same for all pixels, i.e. \( \tilde{R}(s' \rightarrow s) = \bar{R}(s' \rightarrow s) \).

Then the LBC becomes:

\( \tilde{I}(H; r; s) = S_0 \mu_0 e^{-\frac{\gamma_0}{\mu_0}} \bar{\rho}(r) R(s_0 \rightarrow s) + \frac{1}{\pi} \int (\tilde{I}(H; r; \gamma') \bar{\rho} + \tilde{I}(H; \gamma') \bar{\rho}(r)) R(s' \rightarrow s) \mu' ds' \)

(2c)

Apply a Fourier-transform

\[
x(p) = \int_{-\infty}^{+\infty} x(r)e^{ipr} \ dr, \ x(r) = \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} x(p)e^{-ipr} \ dp, \ \text{where} \ p=(p_x, p_y),
\]

and rewrite problem (2) in the Fourier domain

\[
\mu \frac{\partial \tilde{I}(z; p; s)}{\partial z} + [-i \sqrt{1 - \mu^2} (p_x \cos \phi + p_y \sin \phi) + \alpha(z)] \tilde{I}(z; p; s) = \frac{\sigma(z)}{4\pi} \int_{\Omega} \chi(z, \gamma) \tilde{I}(z; \tau; \gamma') ds'
\]

\( \tilde{I}(0; p; s) = 0; \)

\( \tilde{I}(H; p; s) = S_0 \mu_0 e^{-\frac{\gamma_0}{\mu_0}} \bar{\rho}(p) R(s_0 \rightarrow s) + \frac{1}{\pi} \int (\tilde{I}(H; p; \gamma') \bar{\rho} + \tilde{I}(H; \gamma') \bar{\rho}(p)) R(s' \rightarrow s) \mu' ds' \)
This problem is linear in variation of reflectance \( \rho(p) \). So the variation of radiance can be factorized as

\[
\tilde{T}(z; p; s) = \tilde{\rho}(p) \psi(z; p; s)
\]

which gives the problem for function \( \psi(z; p; s) \)

\[
\mu \frac{\partial \psi(z; p; s)}{\partial z} + \left[ -i \sqrt{1 - \mu^2} (p_x \cos \phi + p_y \sin \phi) + \alpha(z) \right] \psi(z; p; s) = \frac{\sigma(z)}{4\pi} \int \chi(z, \gamma) \psi(z; p; s') ds' \quad (4a)
\]

\[
\psi(0; p; s) = 0;
\]

\[
\psi(H; p; s) = S_{\lambda} \mu_0 e^{-\tau_0/\mu_0} R(s_0 \rightarrow s) + \frac{1}{\pi \Omega^*} \int \{ \psi(H; p; s') \tilde{\rho} + \tilde{T}(H; s') \} R(s' \rightarrow s) \mu' ds'
\]

\[
= J_0(H, s) + \frac{1}{\pi \Omega^*} \int \psi(H; p; s') \tilde{\rho} R(s' \rightarrow s) \mu' ds', \quad \mu < 0 \quad (4c)
\]

The mean reflected radiance \( \tilde{T}_0(\tau_0, s) = \tilde{\rho} J_0(H, s) \) is a solution of a 1-D anisotropic problem on the lower boundary in upward direction, and

\[
J_0(H, s) = S_{\lambda} \mu_0 e^{-\tau_0/\mu_0} R(s_0 \rightarrow s) + \frac{1}{\pi \Omega^*} \int \tilde{T}(H; s') R(s' \rightarrow s) \mu' ds' = \tilde{T}_0(\tau_0, s) / \tilde{\rho}, \quad \mu < 0 \quad (5)
\]

Problem (4a-4c) can be solved by iterations. The first iteration is a solution at \( \rho = 0 \), i.e. with LBC

\[
\psi^{(1)}(H; p; s) = J_0(H, s), \quad \mu < 0 \quad (6a)
\]

and LBC for \( n \)th iteration is

\[
\psi^{(n)}(H; p; s) = J_0(H, s) + \frac{1}{\pi \Omega^*} \int \psi^{(n-1)}(H; p; s') \tilde{\rho} R(s' \rightarrow s) \mu' ds' = J_0(H, s) / (1 - \tilde{\rho} c(p)), \quad \mu < 0 \quad (6b)
\]

In the upward directions,

\[
\psi(\tau; p; s) \equiv e^{i\phi(p)} \left\{ e^{-\tau_0/\mu_0} J_0(\tau_0, s) + A(\tau; p; s) e^{i\Phi(\tau, p, s)} \right\}, \quad x_1 = (H-z) \tan \theta \cos \phi \quad (7)
\]

The signal at the top of atmosphere

\[
I(x; z) = \tilde{T}(s) + \tilde{\rho}(x-x_1) J_0(\tau_0, s) e^{-\tau_0/\mu_0} + \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{\rho}(p) A(p, s) e^{-i[p(x-x_1)-\Phi(p, s)]} dp \quad (8a)
\]

Now,

\[
\tilde{\rho}(x-x_1) J_0(\tau_0, s) = S_{\lambda} \mu_0 e^{-\tau_0/\mu_0} \tilde{\rho}(x-x_1; s_0, s) + \frac{1}{\pi \Omega^*} \int \tilde{T}(H; s') \tilde{\rho}(x-x_1; s', s) \mu' ds',
\]

and here the left-hand side is approximate and the right-hand side is exact. Finally,

\[
I(x; z) = \tilde{T}(s) + \left\{ S_{\lambda} \mu_0 e^{-\tau_0/\mu_0} \tilde{\rho}(x-x_1; s_0, s) + \frac{1}{\pi \Omega^*} \int \tilde{T}(H; s') \tilde{\rho}(x-x_1; s', s) \mu' ds' \right\} e^{-\tau_0/\mu_0} + \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{\rho}(p) A(p, s) e^{-i[p(x-x_1)-\Phi(p, s)]} dp \quad (8b)
\]

In order to solve problem (4a-4c) we need only the average BRDF \( \tilde{\rho}(s', s) \). In addition, variations of BRDF \( \tilde{\rho}(r; s_0 \rightarrow s) \) and of albedo \( \tilde{\rho}(x-x_1) \) are required in (8b) to find radiance.
2.2 Determination of mean and variation for the forward modeling

Given: \( \rho(r_{i,j}; s' \rightarrow s) \) is BRDF in the pixel \((i, j)\),

Find: a) \( \overline{\rho}(s' \rightarrow s) \), \( \bar{\rho}(r; s' \rightarrow s) \); b) \( \bar{\rho} \), \( R(s' \rightarrow s) \); c) \( \bar{\rho}(r_{i,j} - x_i) \)

Solution:

a) For all combinations of angles \((\theta', \theta, \phi - \phi')\) find an average and variation of BRDF
\[
\overline{\rho}(s' \rightarrow s) = \frac{1}{N} \sum_{i,j} \rho(r_{i,j}; s' \rightarrow s) ; \quad \bar{\rho}(r_{i,j}; s' \rightarrow s) = \rho(r_{i,j}; s' \rightarrow s) - \overline{\rho}(s' \rightarrow s) \tag{9}
\]

b) Find solution of 1-D problem with BRDF \( \overline{\rho}(s' \rightarrow s) \) for
- \( \overline{I}(s) \), mean radiance at the TOA,
- \( \overline{I}_0(\tau_0, s) \), mean reflected radiance in the upward directions at BOA, and
- \( \overline{\rho} = \frac{F^\dagger(\tau_0)}{F^\dagger(\tau_0)} \), mean surface albedo.

c) Define \( R(s' \rightarrow s) = \overline{\rho}(s' \rightarrow s) / \overline{\rho} \) \tag{10}

d) Calculate value \( \bar{\rho}(r_{i,j}) \) using the law of energy conservation in every pixel, namely: the reflected fluxes from the true \( \bar{\rho}(r_{i,j}; s' \rightarrow s) \) and approximate \( \bar{\rho}(r_{i,j}) \) \( R(s' \rightarrow s) \) BRDF variation should be equal:
\[
\int_{\Omega^-} ds \int_{\Omega^+} I^\dagger(\tau_0, s') \mu \mu' \left\{ \bar{\rho}(r_{i,j}; s' \rightarrow s) - \bar{\rho}(r_{i,j}) \right\} ds' = 0, \Rightarrow \overline{\rho}(r_{i,j}) = \overline{\rho} \int_{\Omega^-} ds \int_{\Omega^+} I^\dagger(\tau_0, s') \mu \mu' \left\{ \bar{\rho}(r_{i,j}; s' \rightarrow s) \right\} ds' \]
\[
= \overline{\rho} \cdot \frac{F^\dagger_{ij}(\tau_0)}{F^\dagger(\tau_0)} \tag{11}
\]

3. Atmospheric correction algorithm

Rewrite formulas (8a-8b):
\[
I(x; s) = \overline{I}(s) + \rho(x - x_i) J_0(\tau_0, s) e^{-\tau_{\mu}/\mu} + \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{\rho}(p) A(p, s) e^{-i p(x - x_i - \Phi(p, s))} dp \tag{8a}
\]
\[
I(x; s) = \overline{I}(s) + \sum \lambda \mu_0 e^{-\tau_{\mu}/\mu_0} \rho(x - x_i; s_0, s) + \frac{1}{\pi} \int_{\Omega} \bar{I}(H; s') \rho(x - x_i; s', s) \mu' ds' e^{-\tau_{\tilde{\mu}}/\mu} + \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{\rho}(p) A(p, s) e^{-i [p(x - x_i - \Phi(p, s))] dp} \tag{8b}
\]

1. Average the radiance over an area \( E_n \) where the view angle may be assumed constant \( \Rightarrow \overline{I}(s) \)

2. Find the variation of radiance \( I(r_{ij}; s) - \overline{I}(s) \) in each pixel \( \Rightarrow (C1) \)

3. We need to estimate the adjacency integral in (8a-8b), in other words to find the albedo variation \( \bar{\rho}(r_{ij} - r_i) \). For this purpose use an approximate Eq. (8a), which assumes that the shape of BRDF is constant.

Apply a Fourier transform to variation \( I(r_{ij}; s) - \overline{I}(s) \):


\[ F[I(r_j; s) - I(s)] = \bar{p}(p) \left\{ J_0(\tau_0, s) e^{-\tau_0/|\mu|} + A(p, s) e^{i\phi(p, s)} \right\} \]

then the adjacency integral is:

\[ \tilde{I}_{ij}^a(s) = F^{-1} \left[ \frac{F[I(r_j; s) - I(s)]A(p, s)e^{i\phi(p, s)}}{J_0(\tau_0; s)e^{-\tau_0/|\mu|} + A(p, s)e^{i\phi(p, s)}} \right] \]

4. Correct the radiance for adjacency effect (use accurate formula (8b)):

\[ I(r_j; s) - \tilde{I}_{ij}^a(s) - I(s) = \{ S_{\lambda} \mu_0 e^{-\tau_0/|\mu|} \bar{p}(r_j - r_i; s_0, s) + \frac{1}{\pi} \int I(H; s') \bar{p}(r_j - r_i; s', s') \mu'ds' \} e^{-\tau_0/|\mu|} \]

Formula C5 is fundamental for BRDF retrieval.

5. Assume, that a set of measurements (composite) is obtained for a number of consequent days \( t_k \), corresponding to different zenith view angles \( s_k \). Then, we have two minimization problems:

a) For mean BRDF:

Solution of 1-D anisotropic problem is:

\[ \bar{I}(s_k) - D(s_k) = I_{reff}(s_k), \text{ where} \]

\[ I_{reff}(s_k) = \{ S_{\lambda} \mu_0 e^{-\tau_0/|\mu|} \bar{p}(s_0, s_k) + \frac{1}{\pi} \int I(H; s') \bar{p}(s', s_k) \mu'ds' \} e^{-\tau_0/|\mu|} + I^{df}(s_k) \]

1. On the 1st iteration find:

\[ \bar{p}^{(1)}(s_0, s_k) = \left[ \bar{I}(s_k) - D(s_k) \right] / S_{\lambda} \mu_0 e^{-\tau_0/|\mu|} \]

2. Calculate the best-fit parameters of MRPV model.

3. Solve the RTE for the TOA \( I_{reff}^{(n-1)}(s_k) \) corresponding to BRDF \( \bar{p}^{(n-1)}(s_0, s_k) \).

4. Find

\[ \bar{p}^{(n)}(s_0, s_k) = \left[ \bar{I}(s_k) - D(s_k) - I_{reff}^{(n-1)}(s_k) - S_{\lambda} \mu_0 e^{-\tau_0/|\mu|} \bar{p}^{(n-1)}(s_0, s_k) \right] / S_{\lambda} \mu_0 e^{-\tau_0/|\mu|} \]

5. Repeat steps 2-4 until \( \bar{I}(s_k) - D(s_k) - I_{reff}(s_k) \leq \varepsilon \) for all \( k \).

This problem is solved only once for every area of averaging \( (E_n) \). Then, for each pixel within this area we solve the problem for the BRDF variation

b) \( I(r_j; s) - \tilde{I}_{ij}^a(s) - I(s) = \{ S_{\lambda} \mu_0 e^{-\tau_0/|\mu|} \bar{p}(r_j - r_i; s_0, s) + \frac{1}{\pi} \int I(H; s') \bar{p}(r_j - r_i; s', s) \mu'ds' \} e^{-\tau_0/|\mu|} \)

The algorithm is:

\[ \bar{p}^{(n)}(r_j - r_i; s_0, s_k) = \left[ I(r_j; s_k) - \tilde{I}_{ij}^a(s_k) - I(s_k) - \frac{1}{\pi} \int I(H; s') \bar{p}^{(n-1)}(r_j - r_i; s', s_k) \mu'ds' \right] e^{-\tau_0/|\mu|} \]

\[ / S_{\lambda} \mu_0 e^{-\tau_0/|\mu|} \]

where the incident radiation \( I(H; s) \) is known from solution of problem a). The integral term in (C6b) can be estimated using numeric quadrature.