Green's function method in the radiative transfer problem. II. Spatially heterogeneous anisotropic surface

Alexei Lyapustin and Yuri Knyazikhin

The most recent theoretical studies have shown that three-dimensional (3-D) radiation effects play an important role in the optical remote sensing of atmospheric aerosol and land surface reflectance. These effects may contribute notably to the error budget of retrievals in a broad range of sensor resolutions, introducing systematic biases in the land surface albedo data sets that emerge from the existing global observation systems. At the same time, 3-D effects are either inadequately addressed or completely ignored in data processing algorithms. Thus there is a need for further development of the radiative transfer theory that can rigorously treat both 3-D and surface anisotropy effects and yet be flexible enough to permit the development of fast forward and inversion algorithms. We describe a new theoretical solution to the 3-D radiative transfer problem with an arbitrary nonhomogeneous non-Lambertian surface. This solution is based on an exact semianalytical solution derived in operator form by the Green's function method. The numerical implementation is based on several parameterizations that accelerate the solution dramatically while keeping its accuracy within several percent under most general conditions. © 2002 Optical Society of America

OCIS codes: 290.4210, 010.1300, 010.1310, 010.1320.

1. Introduction

Optical remote sensing studies of land surface are based on analysis of the surface-reflected signal. The formation of this signal is a complex process that involves multiple interactions of light with an inhomogeneous and anisotropic surface boundary and direct and diffuse transmission of reflected radiance through the atmosphere. Surface inhomogeneity gives rise to horizontal radiative fluxes in the atmosphere directed from the bright to the dark surface areas. These three-dimensional (3-D) effects reduce the apparent top-of-the-atmosphere (TOA) surface contrast by decreasing the radiance over bright pixels and increasing the brightness of the dark pixels. This blurring effect is systematic and thus becomes important for the remote sensing applications developed for use with either dark or bright targets. In recent papers[1,2] it was shown that the 3-D effects described cause a systematic overestimation of the aerosol optical thickness retrieved over land by the dark target method[3] and a nonnegligible systematic error in the land surface albedo at a broad range of sensor resolutions that may exceed the tolerance of climate models in many climatically significant regions marked by medium-to-high surface contrast.[2]

At present, 3-D effects are neglected in aerosol retrieval over land and are either ignored or only approximately addressed in the available atmospheric correction algorithms. Thus, although the physical processes involved in the radiative transfer are well known and have been studied in various degrees of detail, there is a need to develop an analytical theory that combines 3-D and surface anisotropy effects with efficient numerical methods for forward radiance modeling in a realistic environment and for comprehensive atmospheric correction of satellite imagery.

Historically, several approaches to solving the radiative transfer problem with a nonhomogeneous non-Lambertian boundary have been used. One of the early numerical solutions with the Fourier-transform Gauss–Seidel method was obtained by Diner and Martonchik.[4,5] A number of studies were done with Monte Carlo techniques,[6,7] although pre-

A. Lyapustin (alyapustin@pop900.gsfc.nasa.gov) is with the Goddard Earth Sciences and Technology Center, University of Maryland Baltimore County, and NASA Goddard Space Flight Center, Greenbelt, Maryland 20771. Y. Knyazikhin is with the Department of Geography, Boston University, Boston, Massachusetts 02215.

Received 6 February 2002; revised manuscript received 19 June 2002.

0003-6935/02/275600-07$15.00/0

dominantly with the Lambertian surface model. Among the latest developments is the spherical harmonic discrete-ordinates method,[8] which handles the complete 3-D problem with the lateral variability of both surface and atmospheric optical properties.

Below, we present a new study of this important issue. First we obtain an exact semianalytical solution, using the Green's function method along with the method of successive surface interactions. This approach has a solid theoretical heritage[9,10] and is a logical continuation of our earlier research with a one-dimensional (1-D) problem.[11] Second, because the straightforward computations based on the derived formulas are extremely time consuming, we devote considerable attention to methods for numerical acceleration of the solution. The proposed parameterizations permit fast numerical implementation with a high degree of accuracy in calculation that is suitable for remote sensing applications.

2. Operator Solution of the Three-Dimensional Radiative Transfer Problem

Diffuse solar radiation in the horizontally homogeneous atmosphere bounded by a nonuniform non-Lambertian surface is a solution of the following 3-D boundary-value problem:

$$[(\boldsymbol{\nu}\nabla) + \alpha(z)]L(z; r; s) = \frac{\sigma(z)}{4\pi}\int_{\Omega} \chi(z, \gamma)L(z; r; s')\mathrm{d}s' + \frac{S_\lambda}{4}\sigma(z)\chi(z; \gamma_0)\exp[-\tau(z)/\mu_0], \tag{1}$$

$$L(0; r; s) = 0,\ \mu > 0, \tag{1a}$$

$$L(H; r; s) = S_\lambda\mu_0 \exp(-\tau_0/\mu_0)\rho(r; s_0, s) + \frac{1}{\pi}\int_{\Omega^+} L(H; r; s')\rho(r; s', s) \times \mu'\mathrm{d}s',\ \mu < 0. \tag{1b}$$

For definitions of our principal terms, see Appendix A. Below, we follow the notation and terminology of the Green's function technique used in our study of the 1-D problem.[11] Similar derivations can be found in general theoretical studies of the radiative transfer.[9,10] Let us decompose the total signal into path radiance $D(z, s_0, s)$ and surface-reflected radiance $J(r; z; s)$:

$$L(z; r; s_0, s) = D(z; s_0, s) + J(z; r; s). \tag{2}$$

As before,[11] we use the operator form of notation and, specifically, a 3-D differential operator $[\hat{L}_3 = (\boldsymbol{\nu}\nabla) + \alpha(z)]$ and integral operators of atmospheric scattering $\{\hat{S} = [\sigma(z)/4\pi] \int_\Omega \mathrm{d}s'\chi(z, \gamma) \ldots\}$ and of surface reflection $[R_r = (1/\pi) \int_{\Omega^+} \mathrm{d}s'\mu'\rho(r; s', s) \ldots]$. For convenience of notation, reflection operator $\mathbf{R}$ is used without the operator sign. Using these operators, we can rewrite the problem for the surface-reflected radiance as follows:

$$\hat{L}_3 J(z; r; s) = \hat{S}J(z; r; s), \tag{3}$$

$$J_+(0; r) = 0, \quad J_-(H; r) = R_r I_+^0(H) + R_r J_+(H; r). \tag{3a}$$

Here the directions of propagation are indicated by subscripts + and −, which stand for the downward ($\mu > 0$) and upward ($\mu < 0$) directions, respectively. In the lower boundary condition of Eqs. (3a), $I_+^0(H)$ denotes the incident radiance formed by the directly transmitted sunlight and path radiance:

$$I_+^0(H) = \pi S_\lambda \exp(-\tau_0/\mu_0)\delta(s - s_0) + D(H; s_0, s),\ \mu > 0. \tag{4}$$

The solution of problem (3)–(3a) can be represented as a series in different orders of interaction of light with surface:

$$J(z; r; s) = \sum_{k\geq 1} J^{(k)}(z; r; s). \tag{5}$$

The substitution of series (5) into problem (3)–(3a) generates a set of recursive problems for different orders of reflection $J^{(k)}$. Each recursive problem satisfies transport equation (3):

$$\hat{L}_3 J^{(k)}(z; r; s) = \hat{S}J^{(k)}(z; r; s), \tag{6}$$

with boundary conditions of the form

$$J_+^{(k)}(0; r) = 0; \qquad J_-^{(k)}(H; r) = R_r J_+^{(k-1)}(H; r), \tag{6a}$$

where, for generality, we have denoted $J_+^{(0)}(H; r) \equiv I_+^0(H)$.

By definition,[12] solution of problem (6)–(6a) for any kth term $J^{(k)}(z; r; s)$ at atmospheric level z in direction s can be expressed by the 3-D surface Green's function $G_3(z; r - r'; s_1, s)$ and the boundary values of radiance $J^{(k)}(H; r; s_1)$ as

$$J^{(k)}(z; r; s) = \int_{-\infty}^{+\infty} \mathrm{d}r' \int_{\Omega^-} G_3(z; r - r'; s_1, s)J^{(k)} \times (H; r'; s_1)\mathrm{d}s_1. \tag{7}$$

The substitution of Eq. (7) into problem (6)–(6a) shows that the Green's function thus introduced does not depend on the reflective properties of the surface and satisfies a classic searchlight problem originally introduced by Chandrasekhar[13]:

$$\hat{L}_3 G_3 = \hat{S}G_3, \tag{8}$$

$$G_{+3}(0; r - r') = 0, \qquad G_{-3}(H; r - r') = \delta(r - r')\delta(s - s_1). \tag{8a}$$

Lower boundary condition (8a) shows that function $G_3(z; r - r'; s_1, s)$ is an atmospheric response to the unitary monodirectional localized perturbation on its lower boundary. For this reason, function G_3 is also called a point-spread function.

In accordance with definition (7), let us introduce the operator

$$\hat{\Gamma}^3_{z;r}(s) = \int_{-\infty}^{+\infty} \mathrm{d}r' \int_{\Omega^-} \mathrm{d}s_1 G_3(z;\, r - r';\, s_1,\, s). \ldots \quad (9)$$

In the physical sense, operators $\hat{\Gamma}^{3+}_{H;r} \equiv \hat{\Gamma}^3_{H;r}$ ($\mu > 0$, φ) and $\hat{\Gamma}^{3-}_{0;r} \equiv \hat{\Gamma}^3_{z=0;r}$ ($\mu < 0$, φ) represent atmospheric transmittance for radiance in the backward and forward directions, respectively, on illumination of the atmosphere from below. Using these operators, we can establish the relation between successive orders of interaction of light with inhomogeneous surfaces. At the surface level, the downward radiance of reflection order (k), $J^{(k)}_+(H;\, r)$, is a result of atmospheric backscattering of the upward radiance of the same order, $J^{(k)}_-(H;\, r)$:

$$J^{(k)}_+(H;\, r) = \hat{\Gamma}^{3+}_{H;r} J^{(k)}_-(H;\, r).$$

The upward radiance of order ($k + 1$) can be expressed by means of its precursor of order (k) as

$$J^{(k+1)}_-(H;\, r) = R_r J^{(k)}_+(H;\, r) = R_r \hat{\Gamma}^{3+}_{H;r} J^{(k)}_-(H;\, r). \quad (10)$$

Based on this result, the total upward surface-reflected radiance in series (5) becomes

$$J_-(H;\, r) = \sum_{k\geq 1} J^{(k)}_-(H;\, r) = \sum_{k\geq 0} (R_r \hat{\Gamma}^{3+}_{H;r})^k R_r I^0_+(H)$$
$$= (\hat{I} - R_r \hat{\Gamma}^{3+}_{H;r})^{-1} R_r I^0_+(H), \quad (11)$$

where $\hat{I}$ is an identity operator and the last equality is based on the fact that the norm of operator $R_r \hat{\Gamma}^{3+}_{H;r}$ is less than 1.[11]

Formula (11) gives an exact formal solution in operator form for the reflected radiance at the surface level. Numerically, the inverse operator in this formula can be evaluated only as a sum of corresponding series (11). The relationships between successive orders of interaction $k \to k + 1$ are described by the operator $R_r \hat{\Gamma}^{3+}_{H;r}$ [Eq. (10)], which represents the following integral transformation:

$$J^{(k+1)}(H;\, r;\, s) = \frac{1}{\pi} \int_{\Omega^+} \mathrm{d}s' \mu' \rho(r;\, s',\, s) \times \int_{-\infty}^{+\infty} \mathrm{d}r' \int_{\Omega^-} G_3(H;\, r - r';\, s_1,\, s') J^{(k)}(H;\, r';\, s_1) \mathrm{d}s_1. \quad (12)$$

At an arbitrary altitude z in the atmosphere, the surface-reflected radiance in the upward direction can be found from Eq. (7):

$$J_-(z;\, r - r_s) = \hat{\Gamma}^{3-}_{z;r-r_s} (\hat{I} - R_r \hat{\Gamma}^{3+}_{H,r})^{-1} R_r I^0_+(H), \quad (13)$$

where $r_s = [(H - z)|\tan\theta|\cos\varphi;\, (H - z)|\tan\theta|\sin\varphi]$ is a shift in the horizontal coordinate at slant observations.

3. Linearized Solution

An exact solution for radiance [Eq. (13)] accounts for all orders of interaction of light with a spatially variable surface bidirectional reflectance distribution function (BRDF). In further analysis, it is convenient to separate the mean value $[\bar{f}]$ and the spatial variation $[\tilde{f}(r)]$ in a surface BRDF and in radiance. We further analyze Eq. (13) in the linear approximation in BRDF spatial variation. That will allow us to develop the fast parametric solution and use a parametric formula derived earlier for the mean component of radiance that corresponds to the mean BRDF.[11] The linearized solution has sufficiently high accuracy for practical purposes. The nonlinear contribution $\tilde{J}_{\mathrm{NL}}(z;\, r;\, s)$ is typically small. For example, in the case of a Lambertian surface the nonlinear contribution is bounded from above[10] as

$$\tilde{J}_{\mathrm{NL}}(z;\, r;\, s)/\tilde{J}(z;\, r;\, s) \leq \frac{q^{\max} c_0}{1 - \bar{q} c_0}, \quad (14)$$

where $q^{\max}$ is the maximal albedo value in the image and c_0 is a spherical albedo of the atmosphere. In clear-sky conditions, $\tilde{J}_{\mathrm{NL}}(z;\, r;\, s)$ does not exceed several percent of the radiance variation and is even smaller with respect to the total signal.

Let us divide the surface-reflection operator (R_r) into the mean ($\bar{R}$) and the variance ($\tilde{R}_r$) and linearize the surface-reflected radiance [Eq. (11)]:

$$J_-(H;\, r) = \sum_{k\geq 0} ([\bar{R} + \tilde{R}_r] \hat{\Gamma}^{3+}_{H;r})^k [\bar{R} + \tilde{R}_r] I^0_+(H)$$
$$\cong \bar{J}_-(H) + \sum_{k\geq 0} (\bar{R} \hat{\Gamma}^{3+}_{H;r})^k \tilde{R}_r I^0_+(H)$$
$$+ \sum_{k\geq 0} \sum_{l=0}^{k} (\bar{R} \hat{\Gamma}^{3+}_{H;r})^{k-l} \tilde{R}_r \hat{\Gamma}^{3+}_{H;r} (\bar{R} \hat{\Gamma}^{3+}_{H;r})^l \bar{R} I^0_+(H). \quad (15a)$$

In this formula the first term, $\bar{J}_-(H)$, is the mean reflected radiance that corresponds to the mean BRDF. Let us rewrite the third term as

$$\sum_{k\geq 0} \sum_{l=0}^{k} (\bar{R} \hat{\Gamma}^{3+}_{H;r})^{k-l} \tilde{R}_r (\hat{\Gamma}^{1+}_H \bar{R})^l \hat{\Gamma}^{1+}_H \bar{R} I^0_+(H). \quad (15b)$$

Here we have used the fact that 3-D operator $\hat{\Gamma}^{3+}_{H;r}$, when it is applied to mean signal $\bar{R} I^0_+(H)$, turns into a 1-D operator $\hat{\Gamma}^{1+}_H$ related to a 1-D Green's function:

$$G_1(z;\, s_1,\, s) = \int_{-\infty}^{+\infty} G_3(z;\, r - r';\, s_1,\, s) \mathrm{d}r', \quad (16)$$

which obeys the boundary-value problem [Eqs. (9) and (9a)] of Ref. 11. Now series (15b) can be summed analytically as follows: Let us fix index $l = l_0$ ($l_0 = 0, 1, \ldots$) and sum the series in index k from $k = l_0$ to $k = \infty$. When the residual series is summed in index l including the second term of formula (15a), it gives the following expression for the variation of surface-reflected radiance:

$$\tilde{J}_-(H; r) \cong (\hat{I} - \bar{R}\hat{\Gamma}^{3+}_{H;r})^{-1}\tilde{R}_r(\hat{I} - \hat{\Gamma}^{1+}_H\bar{R})^{-1}I^0_+(H). \quad (17)$$

Formula (17) shows that, in the linear approximation, the variation of the surface-reflected radiance is formed as follows: First, a mean incident radiance is formed that includes all orders of interaction with the mean surface BRDF, $(\hat{I} - \hat{\Gamma}^{1+}_H\bar{R})^{-1}I^0_+(H)$. The incident radiance is then reflected from the BRDF variation, $\tilde{R}_r(\hat{I} - \hat{\Gamma}^{1+}_H\bar{R})^{-1}I^0_+(H)$. Finally, the variation of reflected signal is enhanced in the process of subsequent multiple interactions with the mean BRDF.

Applying a spatial Fourier transform, we can write an expression for the spatial Fourier spectrum of radiance variation at altitude z:

$$\tilde{J}_-(z; p) = F[\tilde{J}_-(z; r)] = \hat{\Gamma}^{3-}_{z;p}(\hat{I} - \bar{R}\hat{\Gamma}^{3+}_{H;p})^{-1}\tilde{R}_p \times (\hat{I} - \hat{\Gamma}^{1+}_H\bar{R})^{-1}I^0_+(H). \quad (18)$$

Here operator $\hat{\Gamma}^{3-}_{z;p}$ is defined as

$$\hat{\Gamma}^{3-}_{z;p} = \int_{\Omega^-} \mathrm{d}s_1 G_3(z; p; s_1, s) \ldots, \qquad \mu < 0. \quad (19)$$

The Fourier transform of the point-spread function, $G_{3p} \equiv G_3\ (z; p; s_1, s)$, is called an optical transfer function (OTF) of the atmosphere. We obtain the boundary-value problem for OTF by applying a Fourier transform to the problem for the point-spread function [Eq. (8)]:

$$\hat{L}_{3p}G_{3p} = \hat{S}G_{3p}, \quad (20)$$

$$G_{+3p}(0) = 0; \qquad G_{-3p}(H) = \delta(s - s_1), \quad (20a)$$

where the transformed differential operator is given by $\hat{L}_{3p} = \mu(\partial/\partial z) - i\sqrt{1 - \mu^2}\ (p_x \cos\varphi + p_y \sin\varphi) + \alpha(z)$. In the downward direction the OTF is a purely diffuse function, G^d_{3p}, and it has both direct and diffuse components in the upward direction:

$$G_3(z; p; s_1, s) = \exp(ipr_s)\begin{cases} G^d_3(z; p; s_1, s) & \mu > 0 \\ \exp\{-[\tau_0 - \tau(z)]/|\mu_1|\}\delta(s - s_1) + G^d_3(z; p; s_1, s) & \mu < 0 \end{cases}. \quad (21)$$

Because operator $\hat{L}_{3p}$ is complex, the diffuse component of the OTF is also a complex function and can be expressed by means of its amplitude A and its phase Φ: $G^d_3(z; p; s_1, s) = A(z; p; s_1, s)\exp[i\Phi(z; p; s_1, s)]$.

The total radiance at altitude z is given by

$$L(z; r; s) = D(z; s_0, s) + \bar{J}(z; s_0, s) + F^{-1}[\tilde{J}(z; p; s)]. \quad (22)$$

Let us now consider a linearized solution as applied to a Lambertian surface, which has been rather well studied both in theory[10] and in numerical experiments.[1,14] This exercise will serve a twofold purpose: On the one hand, we shall show that the Lambertian approximation is a particular case of a general solution [Eq. (18)]. On the other hand, based on the Lambertian solution, we introduce important parameterizations that will allow us to significantly accelerate calculations in the general case of anisotropic surface reflectance.

4. Lambertian Approximation

In the case of isotropic surface reflectance, formula (18) is transformed in the following four steps.

(1) The variation of the reflected radiance becomes

$$\tilde{R}_p(\hat{I} - \hat{\Gamma}^{1+}_H\bar{R})^{-1}I^0_+(H) = \tilde{R}_p \sum_{k\geq 0} (\hat{\Gamma}^{1+}_H\bar{R})^k I^0_+(H) = \tilde{q}(p) \sum_{k\geq 0} (c_0\bar{q})^k E_0(\mu_0) = \tilde{q}(p)\alpha E_0(\mu_0), \quad (23a)$$

where $E_0(\mu_0)$ is surface irradiance; $E_0(\mu_0) = S_\lambda\mu_0 \exp(-\tau_0/\mu_0) + (1/\pi)\int_{\Omega^+} D(H; s_0, s)\mu \mathrm{d}s$; $\alpha = (1 - \bar{q}c_0)^{-1}$; and c_0 is the spherical albedo of the atmosphere, $c_0 = (1/\pi)\int_{\Omega^+} \mu'\mathrm{d}s' \int_{\Omega^-} G^d_1(H; s_1, s')\mathrm{d}s_1$.

(2) Operator $\bar{R}\hat{\Gamma}^{3+}_{H;p}$, which couples atmospheric backscattering and reflection from the mean BRDF, modifies the variation of surface-reflected radiance as follows:

$$(\bar{R}\hat{\Gamma}^{3+}_{H;p})\tilde{R}_p(\hat{I} - \hat{\Gamma}^{1+}_H\bar{R})^{-1}I^0_+(H) = \frac{\bar{q}}{\pi}\int_{\Omega^+}\int_{\Omega^-} \mu' G^d_3(H; p; s_1, s')\mathrm{d}s_1\mathrm{d}s' \times \tilde{q}(p)\alpha E_0(\mu_0) = [\bar{q}c(p)]\tilde{q}(p)\alpha E_0(\mu_0), \quad (23b)$$

where $c(p) = (1/\pi)\int_{\Omega^+} \mu'\mathrm{d}s' \int_{\Omega^-} G^d_3(H; p; s_1, s')\mathrm{d}s_1$ is a spherical albedo of the atmosphere at spatial frequency p. At $p \to 0$ it turns into a 1-D spherical albedo of the atmosphere, $c_0 \equiv c(0)$.

Equation (23b) allows us to express the linear variation of surface-reflected radiance as

$$\tilde{J}(H; r; s) = \sum_{k\geq 0} (\bar{R}\hat{\Gamma}^{3+}_{H;r})^k\tilde{R}_p(\hat{I} - \hat{\Gamma}^{1+}_H\bar{R})^{-1}I^0_+(H) = \sum_{k\geq 0} [\bar{q}c(p)]^k \times \tilde{q}(p)\alpha E_0(\mu_0) = \tilde{q}_m(p)\alpha E_0(\mu_0), \quad (23c)$$

where $\tilde{q}_m(p) = \tilde{q}(p)/[1 - \bar{q}c(p)]$ is the Fourier transform of the albedo variation corrected for multiple interactions between the atmosphere and the mean surface albedo.

(3) Because the angular distribution of reflected radiance $\tilde{J}(H; r; s)$ is isotropic, operator $\hat{\Gamma}^{3-}_{z;p}$ becomes a scalar function:

$$\hat{\Gamma}^{3-}_{z;p}(I - \bar{R}\hat{\Gamma}^{3+}_{H;p})^{-1}\tilde{R}_p(\hat{I} - \hat{\Gamma}^{1+}_{H}\bar{R})^{-1}I^0_+(H)$$
$$= \tilde{q}_m(p) \times \alpha E_0(\mu_0)\int_{\Omega^-} G_3(z; p; s_1, s)\mathrm{d}s_1$$
$$= \tilde{q}_m(p)\alpha E_0(\mu_0)\Psi(z; p; s). \qquad (23d)$$

Here

$$\Psi(z; p; s) = \int_{\Omega^-} G_3(z; p; s_1, s)\mathrm{d}s_1$$
$$= \exp(ipr_s)\begin{cases} \int_{\Omega^-} G^d_3(z; p; s_1, s)\mathrm{d}s_1 & \mu > 0 \\ \exp\{-[\tau_0 - \tau(z)]/|\mu|\} + \int_{\Omega^-} G^d_3(z; p; s_1, s)\mathrm{d}s_1 & \mu < 0 \end{cases} \qquad (23e)$$

is the atmospheric OTF for the problem with a Lambertian surface. From Eqs. (23e) and (20) it follows that function $\Psi_p \equiv \Psi(z; p; s)$ obeys the problem with an isotropic unitary source on the lower boundary:

$$\hat{L}_{3p}\Psi_p = \hat{S}\Psi_p, \qquad (24)$$

$$\Psi^+_p(0) = 0, \qquad \Psi^-_p(H) = 1. \qquad (24a)$$

(4) Finally, the inverse Fourier transform of Eq. (23d) gives the radiance variation in the upward direction:

$$\tilde{J}(z; r; s) = \alpha E_0(\mu_0)\Bigg(\tilde{q}_m(r - r_s)\exp\{-[\tau_0 - \tau(z)]/|\mu|\}$$
$$+ \frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty} \tilde{q}_m(p)A(z; p; s)\exp\{-i[p(r$$
$$- r_s) - \Phi(z; p; s)]\}\mathrm{d}p\Bigg), \qquad (25)$$

where A and Φ are the amplitude and the phase, respectively, of the diffuse component of OTF $\Psi(z; p; s)$.

The boundary-value problem [Eqs. (24) and (24a)] that define OTF $\Psi(z; p; s)$ and the solution for the radiance variation (25) were obtained earlier for the linearized radiative transfer problem with a Lambertian lower boundary.[10,14] Our independent derivation thus shows that the Lambertian solution is a particular case of general solution (17).

5. Numerical Aspects

A brief analysis of formula (18) for the radiance variation shows that straightforward calculations would require an exorbitant amount of computer time. The major problems would be the evaluation of multiple reflections from the surface (inverse operator) and the Fourier transforms. We can introduce several parameterizations that accelerate the solution dramatically while keeping the accuracy sufficiently high for most applications.

First we use the maximum eigenvalue method[11] to parameterize multiple reflections in the term

$$\tilde{R}_p(\hat{I} - \hat{\Gamma}^{1+}_{H}\bar{R})^{-1}I^0_+(H) = \tilde{R}_pI^0_+(H) + \tilde{R}_p\sum_{k\geq 1}(\hat{\Gamma}^{1+}_{H}\bar{R})^kI^0_+(H). \qquad (26a)$$

Separating incident radiance $I^0_+(H)$ into direct solar beam I^+_δ and incident path radiance D^+, we can rewrite the last term as [see the derivation of Eqs. (28) and (29) of Ref. 11]

$$\tilde{R}_p\sum_{k\geq 0}(\hat{\Gamma}^{1+}_{H}\bar{R})^k\hat{\Gamma}^{1+}_{H}\bar{R}I^0_+(H) \cong \frac{\tilde{R}_p\hat{\Gamma}^{1+}_{H}\bar{R}(I^+_\delta + D^+)}{1 - \bar{\eta}}, \qquad (26b)$$

where $\bar{\eta}$ is a maximum eigenvalue of operator $\hat{\Gamma}^{1+}_{H}\bar{R}$. Next, following Ref. 11, we can write

$$\tilde{R}_p\hat{\Gamma}^{1+}_{H}\bar{R}I^+_\delta \approx S_\lambda\mu_0\exp(-\tau_0/\mu_0)c_0\tilde{\rho}_1(p; s)\bar{\rho}_2(s_0),$$
$$\tilde{R}_p\hat{\Gamma}^{1+}_{H}\bar{R}D^+ \approx \bar{\eta}\tilde{R}_pD^+, \qquad (26c)$$

where

$$\rho_1(s) = \frac{1}{2\pi}\int_{\Omega^+} \rho(s', s)\mathrm{d}s',$$

$$\rho_2(s_0) = \frac{1}{2\pi}\int_{\Omega^-} \rho(s_0, s)\mathrm{d}s.$$

Evaluation of maximum eigenvalue $\bar{\eta}$ in the Lambertian approximation $\bar{\eta} \approx \bar{q}(\theta_0)c_0$ allows us to rewrite relation (26a) as

$$\tilde{R}_p(\hat{I} - \hat{\Gamma}^{1+}_{H}\bar{R})^{-1}I^0_+(H) \cong S_\lambda\mu_0\exp(-\tau_0/\mu_0)[\tilde{\rho}(p; s_0, s) + \alpha c_0\tilde{\rho}_1(p; s)\bar{\rho}_2(s_0)]$$
$$+ \frac{\alpha}{\pi}\int_{\Omega^+} D(H; s_0, s') \times \tilde{\rho}(p; s', s)\mu'\mathrm{d}s', \qquad (26d)$$

where

$$\alpha = [1 - \bar{q}(\theta_0)c_0]^{-1}. \qquad (26e)$$

Relation (26d) gives the linearized variation of the surface-reflected radiance $\tilde{J}(H; p; s)$ without the contribution of multiple reflections from the term $(\hat{I} - \bar{R}\hat{\Gamma}_{H;p}^{3+})^{-1}$. Forgetting for a moment about this term, and keeping in mind Eqs. (7) and (21), we can express the (TOA) variation of radiance as

$$\tilde{J}(0; r; s) \cong F^{-1}\left\{ \exp(ipr_s)\left[\tilde{J}(H; p; s)\exp(-\tau_0/|\mu|) + \int_{\Omega^-} G_{3p}^d(0; p; s_1, s)\tilde{J}(H; p; s_1)\mathrm{d}s_1 \right]\right\}. \tag{27}$$

A quick analysis of relation (27) shows that the actual Fourier transform of the BRDF variation is a major computational issue. For example, separate direct and inverse fast-Fourier-transform operations are required not only for a given solar angle and view directions but also for all directions of numerical quadrature that are necessary for integration over the hemispheres Ω^+ and Ω^-, which would make this method completely impractical. However, all the burden in practice can be reduced to a single direct and inverse Fourier transform.

To demonstrate this, let us consider the second (diffuse) term in relation (27). In the physical sense, it represents photons collected from a large elliptical area surrounding the pixel $(r - r_s)$. These photons get into the line of sight after a single or several acts of scattering in the atmosphere. In each instance of scattering, these photons lose their memory of the original direction of propagation. The photons also arrive from different surface points, each of which is characterized by its unique bidirectional reflectance function and geometry of reflection. Thus the combination of horizontal and angular averaging as a result of scattering in the atmosphere produces an effective smoothing of the diffuse transmission of the variation of reflected radiance in angular coordinates. In turn, this smoothing allows us to model the diffuse term of relation (27) in the Lambertian approximation [second term of Eq. (25)]. To this end we introduce an effective Lambertian surface with albedo distribution defined from the condition of conservation of the specific reflected flux in each surface point:

$$q(\mu_0; r) = F^{\uparrow}(H; r)/\bar{F}^{\downarrow}(H; \mu_0), \tag{28a}$$

$$\bar{F}^{\downarrow}(H; \mu_0) = \pi E_0(\mu_0) = \pi S_\lambda \mu_0 \exp(-\tau_0/\mu_0) + \int_{\Omega^+} D(H; s_0, s')\mu' \mathrm{d}s', \tag{28b}$$

$$F^{\uparrow}(H; r) = \int_{\Omega^-} \mu \mathrm{d}s \left[S_\lambda \mu_0 \exp(-\tau_0/\mu_0)\rho(r; s_0, s) + \frac{1}{\pi}\int_{\Omega^+} \mu'\rho(r; s', s)D(H; s_0, s')\mathrm{d}s' \right]. \tag{28c}$$

In the next step in eliminating the multiple Fourier transforms we also neglect the multiple reflection term $(\hat{I} - \bar{R}\hat{\Gamma}_{H;p}^{3+})^{-1}$ in the directly transmitted variation of the surface-reflected radiance [the first term of relation (27)]. In the end, using formula (30) of Ref. 11 for the mean surface-reflected radiance at the TOA, we get the final expression for the total TOA radiance:

$$\begin{aligned} L(r - r_s; s_0, s) \cong{}& D(0; s_0, s) \\ &+ \exp(-\tau_0/|\mu|)J(H; r; s) \\ &+ \int_{\Omega^-} G_1^d(0; s_1, s)\bar{J}(H, s_1)\mathrm{d}s_1 \\ &+ \frac{\alpha E_0(\mu_0)}{(2\pi)^2}\int_{-\infty}^{+\infty} \frac{\tilde{q}(\theta_0, p)A(0; p; s)}{1 - \bar{q}(\theta_0)c(p)} \\ &\times \exp\{-i[p(r - r_s) \\ &- \Phi(0; p; s)\}\mathrm{d}p, \end{aligned} \tag{29}$$

where the surface-reflected radiance is given by

$$\begin{aligned} J(H; r; s) \cong{}& S_\lambda\mu_0 \exp(-\tau_0/\mu_0)[\rho(r; s_0, s) \\ &+ \alpha c_0 \rho_1(r; \mu)\bar{\rho}_2(\mu_0)] \\ &+ \frac{\alpha}{\pi}\int_{\Omega^+} D(H; s_0, s')\rho(r; s', s)\mu'\mathrm{d}s'. \end{aligned} \tag{30}$$

Relation (29) shows that the TOA radiance is represented physically as a sum of the path radiance and the surface-reflected radiation that is directly and diffusely transmitted through the atmosphere. The diffusely transmitted component is additionally split into the diffuse transmission of the mean component and of variation of the surface-reflected radiance.

6. Conclusions

In the present and earlier[11] papers we describe a new, fast, semianalytical solution of a 3-D radiative transfer problem with an inhomogeneous anisotropic surface boundary. It is based on the exact operator formula for radiance, derived here by the method of successive surface interactions and the Green's function technique. The accuracy of formula obtained for the TOA radiance is a result of the interplay of several approximations. First, we neglected the nonlinear component of the spatial variation of radiance. Second, we parameterized multiple interactions of light with the mean BRDF, using a Lambertian formula. Third, we used a Lambertian approximation for the diffusely transmitted variation of the surface-reflected radiance. Extensive numerical study of the solution by intercomparison[15] with a rigorous 3-D code SHDOM[8] showed that introduced approximations work well and in accordance with theoretical predictions, resulting in an overall accuracy within several percent under general conditions. Its high speed and accuracy make the solution described attractive for remote sensing applications.

The analytical form is one of the great advantages of the new solution, which allows us not only to optimize the forward radiance calculations but also to develop rigorous inversion algorithms, such as atmospheric correction. It should be mentioned that, though the current algorithm does not take polarization into account, the theoretical Green's function solution holds for both scalar and vector cases.

Numerical implementation of relations (29) and (30) is based on calculations of the path radiance and the 1-D Green's function with the 1-D spherical harmonics code SHARM originally developed by Muldashev *et al.*[16] The amplitude and the phase of the OTF are calculated with our recently developed spherical harmonics method.[14] These supplementary calculations have the accuracy of a high-order spherical harmonics solution. Thus, with all the atmospheric functionals available, we can generate realistic 3-D radiance fields for any surface and for arbitrary optical conditions of the horizontally homogeneous atmosphere.

Appendix A. Principal Notation Used in This Paper

$\sigma(z)$, $\alpha(z)$, $\tau(z)$, and $\omega(z)$ Scattering and extinction coefficients of the atmosphere and optical thickness and single-scattering albedo at altitude z, respectively

$\chi(z, \gamma)$ Atmospheric scattering function, normalized by 4π (γ is the angle of scattering)

$s = (\theta, \varphi)$ Direction of propagation, defined by zenith and azimuthal angles θ and φ

$\boldsymbol{\nu} = (\sqrt{1 - \mu^2} \cos \varphi, \sqrt{1 - \mu^2} \sin \varphi, \mu)$ Vector of direction, $\mu = \cos \theta$

$\boldsymbol{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$ Vector of gradient

$\rho(r; s', s)$ Surface BRDF at a horizontal coordinate $r = (x, y)$

S_λ Extraterrestrial solar irradiance at wavelength λ

$J(z; r; s)$ Surface-reflected radiance at point (x, y, z) in the direction s

$F[y(r)] = y(p) = \int_{-\infty}^{+\infty} y(r)\exp(ipr)\mathrm{d}r$ and $F^{-1}[y(p)] = y(r) = [1/(2\pi)^2] \int_{-\infty}^{+\infty} y(p)\exp(-ipr)\mathrm{d}p$ Direct and inverse Fourier transforms, where $p = (p_x, p_y)$ is the spatial frequency

$c(p)$ Spherical albedo of the atmosphere at spatial frequency p, $c_0 = c(0)$

$\Psi(z; p; s)$ OTF of the atmosphere

$A(z; p; s)$ and $\Phi(z; p; s)$ Amplitude and phase of the diffuse OTF

For other definitions and terms, see Ref. 11.

References

1. A. I. Lyapustin, "3-D effects in the remote sensing of surface albedo," IEEE Trans. Geosci. Remote Sens. **39,** 254–263 (2001).
2. A. I. Lyapustin and Y. J. Kaufman, "The role of adjacency effect in the remote sensing of aerosol over land," J. Geophys. Res. **106,** 11,909–11,916 (2001).
3. Y. J. Kaufman, D. Tanre, L. A. Remer, E. F. Vermote, A. Chu, and B. N. Holben, "Operational remote sensing of tropospheric aerosol over land from EOS moderate resolution imaging spectroradiometer," J. Geophys. Res. **102,** 17,051–17,068 (1997).
4. D. J. Diner and J. V. Martonchik, "Atmospheric transfer of radiation above an inhomogeneous non-Lambertian reflective ground. I. Theory," J. Quant. Spectrosc. Radiat. Transfer **31,** 97–125 (1984).
5. D. J. Diner and J. V. Martonchik, "Atmospheric transfer of radiation above an inhomogeneous non-Lambertian reflective ground. II. Computational considerations and results," J. Quant. Spectrosc. Radiat. Transfer **32,** 279–304 (1984).
6. W. A. Pearce, "A study of the effect of atmosphere on the Thematic Mapper observations," Rep. 004-77 (EG&G/Washington Analytical Services Center, Inc., Riverdale, Md., 1977).
7. G. Marchuk, G. Mikhailov, N. Nazaraliev, R. Darbinjan, B. Kargin, and B. Elepov, *The Monte Carlo Methods in Atmospheric Optics* (Springer-Verlag, New York, 1980).
8. K. F. Evans, "The spherical harmonics discrete ordinate method for three-dimensional atmospheric radiative transfer," J. Atmos. Sci. **55,** 429–446 (1998).
9. T. A. Germogenova, *The Local Properties of the Solution of the Transport Equation* (Nauka, Moscow, 1986; in Russian).
10. T. A. Sushkevich, S. A. Strelkov, and A. A. Ioltuhovskii, *Method of Path Integration in the Problems of Atmospheric Optics* (Nauka, Moscow, 1990; in Russian).
11. A. I. Lyapustin and Yu. Knyazikhin, "Green's function method in the radiative transfer problem. I. Homogeneous non-Lambertian surface," Appl. Opt. **40,** 3495–3501 (2001).
12. K. M. Case and P. F. Zweifel, *Linear Transport Theory* (Addison-Wesley, Reading, Mass., 1967).
13. S. Chandrasekhar, "On the diffuse reflection of a pencil of radiation by a plane-parallel atmosphere," Proc. Natl. Acad. Sci. USA **44,** 933–940 (1958).
14. A. I. Lyapustin and T. Z. Muldashev, "Solution for atmospheric optical transfer function using spherical harmonics method," J. Quant. Spectrosc. Radiat. Transfer **68,** 43–56 (2001).
15. A. I. Lyapustin, "Radiative transfer code SHARM-3D for radiance simulations over non-Lambertian nonhomogeneous surface: intercomparison study," Appl. Opt. **41,** 5607–5615 (2002).
16. T. Z. Muldashev, A. I. Lyapustin, and U. M. Sultangazin, "Spherical harmonics method in the problem of radiative transfer in the atmosphere–surface system," J. Quant. Spectrosc. Radiat. Transfer **61,** 393–404 (1999).