Chapter III Radiative Transfer in Vegetation Canopies

III.1. The Radiative Transfer Equation for Vegetation Canopies

Solar radiation scattered from a vegetation canopy and measured by satellite sensors results from interaction of photons traversing through the foliage medium, bounded at the bottom by a radiatively participating surface. Therefore to estimate the canopy radiation regime, three important features must be carefully formulated. They are (1) the architecture of individual plant and the entire canopy; (2) optical properties of vegetation elements (leaves, stems) and soil; the former depends on physiological conditions (water status, pigment concentration); and (3) atmospheric conditions which determine the incident radiation field [*Ross*, 1981]. Photon transport theory aims at deriving the solar radiation regime, both within the vegetation canopy and the radiant exitance, using the above mentioned attributes as input data. The leaf area density distribution, u_{L}, leaf normal orientation distribution, g_{L}, leaf scattering phase function, γ_{L}, and boundary conditions specify these input.

Let the domain V in which a vegetation canopy is located, be a parallelepiped of horizontal dimensions X_{S}, Y_{S}, and height Z_{S}. The top δV_{t}, bottom δV_{b}, and lateral δV_{l} surfaces of the parallelepiped form the canopy boundary $\delta V=\delta V_{\mathrm{t}}+\delta V_{\mathrm{b}}+\delta V_{\mathrm{l}}$. Note the boundary δV is excluded from the definition of V. The function characterizing the radiative field in V (i.e., within the canopy space) is the specific intensity introduced by Eq. (1.2). Under condition of the absence of polarization, frequency shifting interaction, and emission processes within the canopy, the monochromatic specific intensity $I_{\lambda}(r,\Omega)$ is given by the steady–state radiative transfer equation (1.21) with $S(r,\Omega)=0$. Substituting (2.13) and (2.17) into the transport equation (1.2), we obtain the radiative transfer equation for vegetation canopies, namely,

$$\Omega \bullet \nabla I_{\lambda}(r,\Omega) + G(r,\Omega)u_{\mathrm{L}}(r)I_{\lambda}(r,\Omega) = \frac{u_{\mathrm{L}}(r)}{\pi}\int_{4\pi}\Gamma_{\lambda}(r,\Omega' \to \Omega)I_{\lambda}(r,\Omega')d\Omega' \quad . \qquad (3.1)$$

The solution to this equation, i.e., the monochromatic specific intensity $I_{\lambda}(r,\Omega)$, depends on wavelength, λ=, location r, and direction Ω. Here, the position vector r denotes the triplet (x,y,z) with $(0<x<X_S)$, $(0<y<Y_S)$ and $(0<z<Z_S)$ and is expressed in Cartesian coordinates with its origin, $O=(0,0,0)$, at the top of the vegetation canopy and the Z axis directed down into the vegetation canopy. The unit vector $\Omega\sim(\mu,\varphi)$ has an azimuthal angle φ measured in the (XY) plane from the positive X axis in a counterclockwise fashion and a polar angle $\theta=\cos(\mu)$ with respect to the polar axis that is opposite to the Z axis. $\Omega \bullet \nabla I_{\lambda}(r,\Omega)$ is a derivative at r along the direction Ω that is defined in section I.5. We shall omit the sign λ denoting the wavelength dependence in notations.

Remark 1. Some measuring instruments (e.g., LICOR quantum sensor) register the radiation fluxes in μmol m^{-2} s^{-1}. Therefore, it is also conventional to use the photon distribution function, $J_{\lambda}(r,\Omega)$ (in mol m^{-3} s^{-1} sr^{-1}), instead of the specific intensity $I_{\lambda}(r,\Omega)$ (in W m^{-3} sr^{-1}). They are related by $I_{\lambda}(r,\Omega)=(hc/\lambda)N_{A}J_{\lambda}(r,\Omega)$ where $h\nu=hc/\lambda$ is the energy of one photon (in J); $h=6.626176\cdot10^{-34}$ J s (Plank constant); $c=2.99792458\cdot10^{8}$ m s^{-1} (the vacuum speed of light); and $N_{A}=6.02205\cdot10^{23}$ mol^{-1} (Avogadro constant); $\lambda=c/\nu$ is the wavelength. Eqs. (1.2) and (3.1) are also valid for the photon density, but its unit is expressed in mol m^{-3} s^{-1} sr^{-1}. Note that $c^{-1}N_{A}J_{\lambda}(r,\Omega)$ is the particle distribution function introduced in section I.1.

Question: the frequency of red light is $\nu=4.3\times10^{14}$ oscillations per second. What is a wavelength λ of red light?

The hemispherical-directional reflectance factor (HDRF) for nonisotropic incident radiation is the ratio of the mean radiance leaving the top of the plant canopy to radiance reflected from an ideal Lambertian target into the same beam geometry and illuminated under identical atmospheric conditions [*Diner et al.*, 1998]. This variable can be expressed in terms of the solution of the radiative transfer equation as

$$R = \frac{\int_0^{X_S}\int_0^{Y_S} I(x,y,0,\Omega)\,dx\,dy}{\frac{1}{\pi}\int_{2\pi-}|\mu'|\,d\Omega'\int_0^{X_S}\int_0^{Y_S} I(x,y,0,\Omega')\,dx\,dy} = \frac{\langle I(\Omega)\rangle_0}{\frac{1}{\pi}\int_{2\pi-}\langle I(\Omega)\rangle_0\,d\Omega'}, \quad \mu > 0\,. \tag{3.2}$$

Here μ and μ' are the cosine of the polar angles of the upward Ω downward Ω', respectively; the angle brackets $< >_0$ denotes the mean over the upper surface δV_t of the parallelepiped V. The HDRF depends on the angular distribution of incoming radiation, the area of the upper boundary δV_t, the height Z_S and the direction Ω. In remote sensing, the dimension of the upper boundary δV_t is called resolution; the upward direction Ω is the view direction. For the condition of no atmosphere, i.e., the incident solar radiation at the upper canopy boundary δV_t is a parallel beam of light, the HDRF is termed a bidirectional reflectance factor (BRF). We use the symbol R_0 to denote its value. The bidirectional reflectance distribution function (BRDF) is a factor of π smaller than BRF, i.e., $\pi^{-1}R_0$.

The bihemispherical reflectance (BHR) for nonisotropic incident radiation is the ratio of the mean radiant exitance to the incident radiant [*Diner et al.*, 1998], i.e.,

$$A = \frac{\int_{2\pi+} < I(\Omega) >_0 |\mu|\,d\Omega}{\int_{2\pi-} < I(\Omega') >_0 |\mu'|\,d\Omega'} \quad . \tag{3.3}$$

The BHR does not depend on the view direction. However, it depends on the incident radiation and the size of the domain V. Note that the BHR is also termed as albedo. For the condition of no atmosphere, the BHR becomes directional hemispherical reflectance (DHR). We reserve symbol A_0 to denote its value.

Problem 3.1: Evaluate the BRF and DHR for a mirror.

Single scattering albedo, $\omega(r,\Omega)$, at the spatial point r and in the direction Ω is the probability that photon travelling along the direction Ω will be scattered by the point r. This probability is the ratio of the scattering coefficient to the extinction coefficient (See section I.2); that is,

$$\omega(r,\Omega)=\frac{\overbrace{\sigma_s'(r,\Omega)}^{\text{substitue Eqs. (1.6) and (2.17)}}}{\underbrace{\sigma(r,\Omega)}_{\text{substitute Eq. (2.13)}}}=\frac{u_{\mathrm{L}}(r)}{u_{\mathrm{L}}(r)}\cdot\frac{\pi^{-1}\int_{4\pi}\Gamma(r,\Omega\to\Omega')\,d\Omega'}{G(r,\Omega)} \tag{3.4}$$

If $u_{\mathrm{L}}(r) \neq 0$ at r, one can cancel u_{L} in Eq. (3.4). The condition $u_{\mathrm{L}}(r) = 0$ means that there are no phytoelements in an elementary volume about the point r and thus the probability of the scattering event is zero, i.e., $\omega(r,\Omega) = 0$. Single scattering albedo can take values between 0 and 1. The vegetation canopy is said to be absorbing medium if $\omega(r,\Omega) = 0$ at any spatial point $r \in V$ for which $u_{\mathrm{L}}(r) \neq 0$ and in any direction Ω. In the case of conservative scattering, $\omega(r,\Omega) = 1$ at any point $r \in V$ for which $u_{\mathrm{L}}(r) \neq 0$ and in any direction Ω.

Problem 3.2: Explain why the condition "for which $u_{\mathrm{L}}(r) \neq 0$" can not be omitted in the definition of the absorbing medium.

Problem 3.3: Show that single scattering albedo satisfies the inequality $\omega(r,\Omega) \leq \varpi$. Here

$$\varpi=\sup_{r\in V;\Omega';\Omega_{\mathrm{L}}}\left\{\rho_{\mathrm{L}}^{-}(r,\Omega',\Omega_{\mathrm{L}})+\tau_{\mathrm{L}}^{-}(r,\Omega',\Omega_{\mathrm{L}});\quad \rho_{\mathrm{L}}^{+}(r,\Omega',\Omega_{\mathrm{L}})+\tau_{\mathrm{L}}^{+}(r,\Omega',\Omega_{\mathrm{L}})\right\}\ , \tag{3.5}$$

where ρ^{-}_{L}, ρ^{+}_{L}, τ^{-}_{L} and τ^{+}_{L} are defined in Eq. (2.7). It follows from definition that $\omega(r,\Omega) \leq 1$. Can ϖ_0 take on a value which exceeds 1? Find ϖ_0 for the purely absorbing medium.

Problem 3.4: Let $\omega_0(r,\Omega)$ be

$$\omega_0(r,\Omega)=\frac{u_{\mathrm{L}}(r)}{u_{\mathrm{L}}(r)}\cdot\frac{\pi^{-1}\int_{4\pi}\Gamma(r,\Omega'\to\Omega)\,d\Omega'}{G(r,\Omega)}\ . \tag{3.6}$$

Note that the integration of Γ is performed over incident directions. Prove the equality $\omega_0(r,\Omega) = \omega(r,\Omega)$ for the bi−Lambertian phase function (see section II.4). Show that this property loses its validity in the general case of leaf normal distribution.

The maximum optical depth $\tau_0(V)$ of the vegetation canopy located in the domain V is defined as

$$\tau_0(V)=\sup_{r,r'\in V}\tau(r,r') \tag{3.7}$$

Here $\tau(r,r')$ is defined in (2.25).

The following theorem is a special case of Germogenova's maximum principle which is proved here under assumption of rotational invariance for the area scattering phase function Γ. If Γ is not rotationally invariant, the condition $\omega_0(r,\Omega) \leq 1$ must hold true to maintain its validity.

Theorem 1. *Let $I(r,\Omega)$ satisfies Eq. (3.1) in the domain V and $\varpi \leq 1$ $\tau_0(V) < \infty$. The following inequality holds true*

$$|I(r,\Omega)|\leq\sup_{r\in\delta V;\,\Omega\bullet\mathbf{n}(r)<0}\left|I(r,\Omega)\right|\ . \tag{3.8}$$

Here $\mathbf{n}(r)$ *is the outward normal to the boundary* δV *at the point* $r \in \delta V$.

This theorem states that the intensity within the vegetation canopy can not exceed a maximum value of the intensity of radiation penetrating into the canopy through the boundary δV. This theorem also presupposes that the incident radiation field is described by a bounded function. It means that this theorem can not be directly applied if the incident radiation is a delta function.

Proof. Let $\bar{I}=\sup_{r\in V;\Omega\in 4\pi}|I(r,\Omega)|$ where "supremum" is taken over all spatial points in V and over all directions. We have

$$\begin{aligned}\Omega\bullet\nabla I_\lambda(r,\Omega) &= -G(r,\Omega)u_{\mathrm{L}}(r)I_\lambda(r,\Omega)+\frac{u_{\mathrm{L}}(r)}{\pi}\int_{4\pi}\Gamma_\lambda(r,\Omega'\to\Omega)I_\lambda(r,\Omega')d\Omega' \\ &\le -G(r,\Omega)u_{\mathrm{L}}(r)I_\lambda(r,\Omega)+\frac{u_{\mathrm{L}}(r)}{\pi}\int_{4\pi}\Gamma_\lambda(r,\Omega'\to\Omega)\underbrace{\sup_{r\in V;\Omega'\in 4\pi}\left\{I_\lambda(r,\Omega')\right\}}_{\text{this term is constant and equal to }\bar{I}}\,d\Omega' \\ &\le -G(r,\Omega)u_{\mathrm{L}}(r)I_\lambda(r,\Omega)+\underbrace{\bar{I}\varpi G(r,\Omega)u_{\mathrm{L}}(r)}_{\substack{\text{use (3.4) and the result of Problem 3.3;}\\ \text{assumption of rotational invariance is}\\ \text{critical to obtain this term}}}\le\underbrace{[\bar{I}-I(r,\Omega)]}_{\text{use the inequality }\varpi\le 1}u_{\mathrm{L}}(r)G(r,\Omega)\ .(3.9)\end{aligned}$$

Comparing the first and last term in (3.9), one can obtain

$$[\bar{I}-I(r,\Omega)]u_{\mathrm{L}}(r)G(r,\Omega)+\underbrace{\Omega\bullet\nabla[\bar{I}-I(r,\Omega)]}_{\text{since }\bar{I}\text{ is constant, }\Omega\bullet\nabla I=-\Omega\bullet\nabla[\bar{I}-I]}\ge 0\ . \qquad (3.10)$$

Multiplying the above by exp(−$\tau(r,r-\xi\Omega,\Omega)$),

$$\tau(r,r-\xi\Omega,\Omega)=\int_0^\xi u_L(r-\xi'\Omega)G(r-\xi'\Omega,\Omega)\,d\xi'\ ,$$

yields

$$-\frac{d}{d\xi}\left\{\left[\bar{I}-I(r-\xi\Omega,\Omega)\right]\exp(-\tau(r,r-\xi\Omega,\Omega)\right\}\ge 0\ \ .$$

Integrating the above over the interval [0,ξ] results in

$$\left[\bar{I}-I(r-\xi\Omega,\Omega)\right]\exp(-\tau(r,r-\xi\Omega,\Omega)\le\bar{I}-I(r,\Omega)\ \ . \qquad (3.11)$$

Let us assume that the solution $I(r,\Omega)$ reaches its maximum at a point r_0 within V and in direction Ω_0, i.e., $\bar{I}=I(r_0,\Omega_0)$. Let ξ_{B} be the distance between the point r_0 and the boundary δV along the direction $-\Omega_0$ opposite to Ω_0. It follows from (3.11) that

$$0\le \underbrace{\underbrace{[I(r_0,\Omega_0)}_{\overset{\parallel}{I}}-I(r_0-\xi_{\mathrm{B}}\Omega_0,\Omega_0)]}_{\text{positive since } I(r_0,\Omega_0) \text{ is the maximum of } I(r,\Omega)}\ \underbrace{\exp(-\tau(r_0,r_0-\xi_{\mathrm{B}}\Omega_0,\Omega_0)}_{\substack{\vee\\ 0\\ \text{because } \tau_0(v)<\infty}}\le \underbrace{I(r_0,\Omega_0)}_{\overset{\parallel}{I}}-I(r_0,\Omega_0)=0\,,$$

which holds true if and only if $I(r_0,\Omega_0) = I(r_0 - \xi_{\mathrm{B}}\Omega_0,\Omega_0)$. It means that the maximum of the solution $I(r,\Omega)$ taken over all internal point and over all directions can not exceed intensity of radiation entering the canopy in the direction Ω_0 through the point r_0 on the boundary δV. *This completes the proof.*

Problem 3.5: Let a vegetation canopy located in the parallelepiped V is isotropically illuminated from above and bounded from below and lateral sides by a black surface. Prove that the BHR is less than 1.

Problem 3.6: Prove that the BHR is an increasing function with respect to single scattering albedo. Do not use the assumptions of the Problem 3.5.

Inequality (3.11) for a more general case was originally derived by *Germogenova* [1986]. This results provides theoretical justification to many existing radiation models. Based on Theorem 1, the following uniqueness theorem can be easily proved under assumption of rotational invariance for the area scattering phase function Γ. We will show in *Remark 2* that this restriction can be relaxed.

Uniqueness Theorem 1. *Let* $\varpi \le 1$ *and* $\tau_0(V) < \infty$. *The radiative regime within a given volume* V *of space bounded by a non-reflecting surface* δV *is uniquely determined by radiation incident on* δV.

Proof. Let $I_1(r,\Omega)$ and $I_2(r,\Omega)$ be two solution of the transport equation subject to given conditions. Writing $\psi(r,\Omega) = I_1(r,\Omega) - I_2(r,\Omega)$ we see that in V, ψ takes on value zero on the boundary δV (and, therefore is a bounded function) and obeys equation (3.1). It follows from Theorem 1 that $|\psi(r,\Omega)| \le 0$. This can be if and only if $\psi(r,\Omega) = 0$, i.e., $I_1(r,\Omega) = I_2(r,\Omega)$. *The uniqueness theorem is proved.*

Problem 3.7: Prove the uniqueness theorem for the transport equation with the emission sources $Q(r,\Omega)$ within V,

$$\Omega\bullet\nabla I(r,\Omega)+G(r,\Omega)u_{\mathrm{L}}(r)I(r,\Omega)=\frac{u_{\mathrm{L}}(r)}{\pi}\int\limits_{4\pi}\Gamma(r,\Omega'\to\Omega)I(r,\Omega')d\Omega'+Q(r,\Omega)\ .$$

<u>Remark 2.</u> Theorem 1 and consequently the uniqueness theorem are proved under assumption of rotational invariance for the area scattering phase function Γ. To extend its validity to the general case of distributed leaf normal, consider an adjoint formulation of the transport equation, i.e.,

$$-\Omega\bullet\nabla I^{*}(r,\Omega)+G(r,\Omega)u_{\mathrm{L}}(r)I^{*}(r,\Omega)=\frac{u_{\mathrm{L}}(r)}{\pi}\int\limits_{4\pi}\Gamma_{\lambda}(r,\Omega\to\Omega')I^{*}(r,\Omega')d\Omega'\ . \qquad (3.12)$$

Note that the integration of the area scattering phase function Γ is performed over scattering angles. Consider the function $I_0^{*}(r,\Omega)=I^{*}(r,-\Omega)$. This function satisfies the equation

$$\Omega \bullet \nabla I_0^*(r,\Omega) + G(r,-\Omega)u_{\mathrm{L}}(r)I_0^*(r,\Omega) = \frac{u_{\mathrm{L}}(r)}{\pi}\int_{4\pi}\Gamma_\lambda(r,-\Omega \to -\Omega')I_0^*(r,\Omega')d\Omega' \ . \qquad (3.13)$$

Theorem 1 can be applied to the above equation using conditions $\varpi \le 1$ *and* $\tau_0(V) < \infty$ without assuming the rotational invariance for Γ. According to Fredholm alternative [*Bronshtein and Semendyaev*, 1985, p. 783], a linear operator equation and its adjoint formulation have a unique solution simultaneously. Therefore, we can use the adjoint transport equation to find conditions under which it has a unique solution. The same conditions guarantee the uniqueness of the transport equation. Thus, the assumption of rotational invariance for the area scattering phase function Γ can be relaxed [*Knyazikhin*, 1990].

Problem 3.8: Prove Theorem 1 and the uniqueness theorem without assuming the rotational invariance for Γ.

Juhan Ross, the founder of the theory of radiative transfer in vegetation canopies, wrote [*Ross*, 1981, p. 144] that in deriving the radiative transfer equation we proceeded from a few contradictory assumptions. On the one hand the assumed elementary volume is so small that no mutual shading exists in any direction within it. On the other hand the number of plates (scatters) in the elementary volume was assumed to be so great that the functions u_{L} and $(1/2\pi)g_{\mathrm{L}}$ are realized to the necessary degree of accuracy. The radiation intensities for which the transfer equation is set down are essentially mean values for the elementary volume. The use of the transport equation, therefore, requires an accurate specification of the elementary volume which depends on the problem one investigates. By other words, the transport equation must be adjusted for a certain application. It can be performed as follows.

Consider a vegetation canopy shown in Figure 3.1. Let the domain V in which the sample site (a square in Figure 3.1) is located be a parallelepiped limited by the slope, a plane parallel to the slope at the height Z_{S} of the tallest tree and a lateral surface of horizontal dimensions X_{S} and Y_{S}. We impose a fine spatial cell mesh of the resolution $\varepsilon\times\varepsilon\times\varepsilon$ on the domain V (Figure 3.1c). Let us segregate a cell (volume) $\Delta V(r)$ around a point r within the stand (Figure 3.1c) so that this cell contains phytoelements without considerable mutual shading in any direction. Evaluate the area $\Delta S(r)$ of leaves within $\Delta V(r)$. Specify mean leaf scattering phase function $\gamma_{\mathrm{L}}(\Delta V)$ for $\Delta V(r)$. Divide the leaf surface within $\Delta V(r)$ into small elementary sub-cells and define the law of distribution of their normals by $g_{\mathrm{L}}(\Delta V,r)$. Displacing the volume $\Delta V(r)$ in any direction within the stand we obtain the values of $u_{\mathrm{L}}(r) = \Delta S(r)/\Delta V(r)$ as well as the functions $g_{\mathrm{L}}(\Delta V,r)$ and $\gamma_{\mathrm{L}}(\Delta V)$. Find a solution, $I(r, \Omega)$, to the transport equation (3.1.) using these variables. The predicted intensity, $I(r, \Omega)$, is a function of the cell size ΔV.

Suppose we have an accurate measurement of the intensity at each fine cell (Figure 3c) and in any direction, i.e., we have $\tilde{I}(r,\Omega)$ derived from field measurements (which does not depends on ΔV). Predicted radiation in one cell can greatly differ from the measured one, e.g., due to mistakes in input for the transport equation. Because the transport equation assumes the energy balance for any elementary volume, some other cells in the neighborhood may exist to compensate for this difference. So the mean predicted intensities over cells from this neighborhood will, much better, agree with the mean measured intensities over the same neighborhood. By other word, the mean predicted and measured values over coarser cells will agree better. These coarser cells specify a resolution at which the radiative transfer equation predicts the radiation field and accuracy of the predictions.

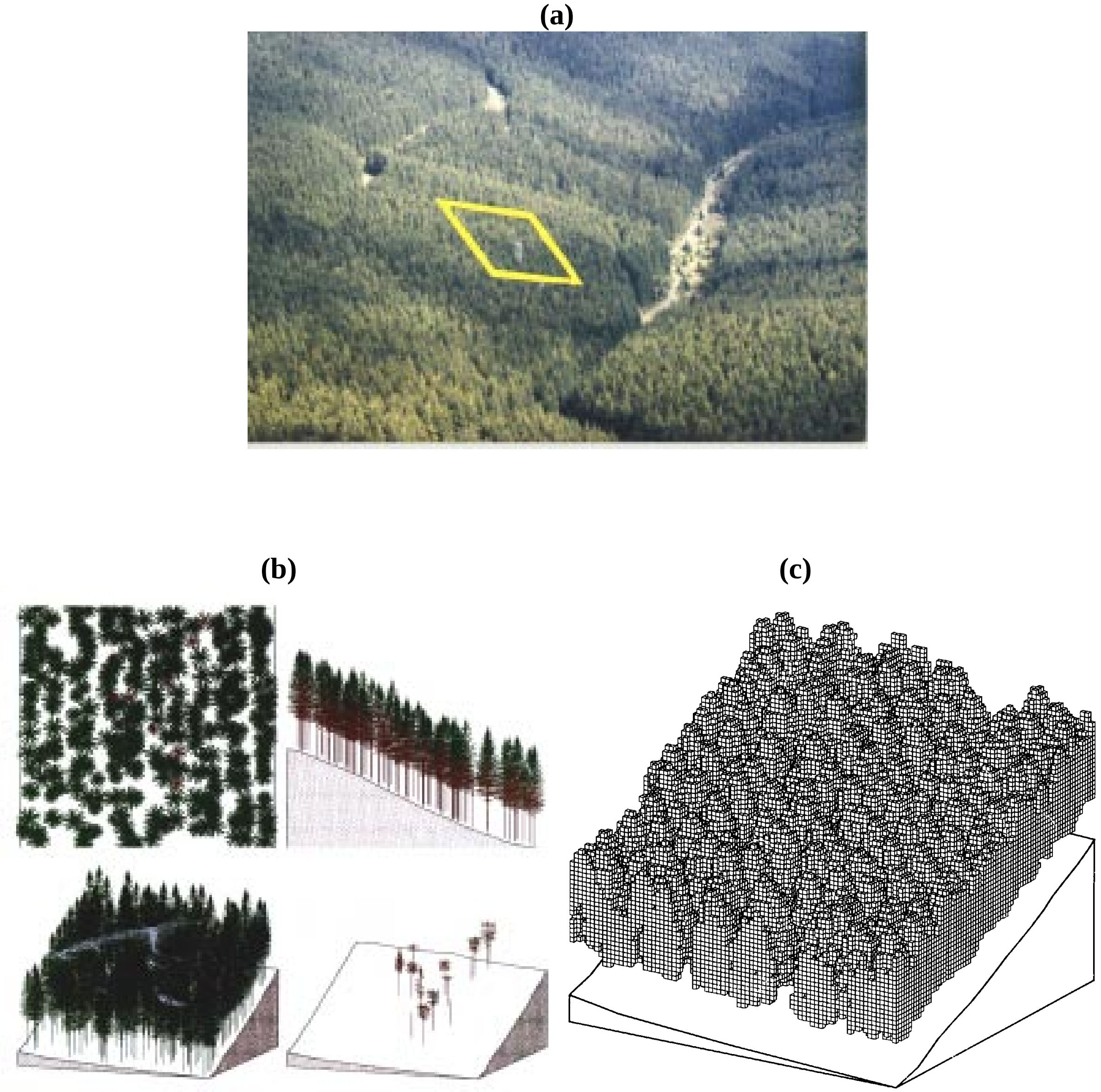

Figure 3.1. (a) Photo shows a Norway spruce stand about 50 km of Goettingen in the Harz mountains. For the sake of detailed examination of the canopy structure, a site covering an area of approximately 40×40 m^2 was chosen (shown as a square) and taken as representative for the whole stand. The forest is about 45 years old and situated on the south slope. There are in total 297 trees in the sample stand. A height of the tallest tree was 12.5 m. The stem diameters varied from 6 to 28 sm. The stand is rather dense but with some local gaps. (b) For the needs of modeling the trees were divided into five groups with respect to the stem diameter. A model of a Norway spruce based on fractal theory was then used to build a representative of each group [*Kranigk and Gravenhorst*, 1993; *Knyazikhin et al*., 1996]. Given tree stem distribution as well as the diameter of each tree, architecture of the entire sample site was generated. (c) Three-dimensional distribution of foliated cells. A fine spatial mesh of the resolution of 50×50×50 cm was imposed on the computer generated sample site and the leaf area density in each of the fine cells was evaluated. The canopy space is limited by the slope and a plane parallel to the slope at the height of 12.5 m. This space contains 160,000 fine mesh sells. Only foliated cells are depicted in this plot.

Thus, we use two attributes to describe the accuracy with which the radiative transfer equation predicts the radiative regime. They are the fine and coarse cells. The first one is used to set down the transfer equation, i.e., to specify u_L, g_L and γ_L at the fine cell and to find a solution, $I(r, \Omega)$, to the transport equation at this resolution. The coarse cell specifies a domain over which the solution $I(r, \Omega)$ must be averaged to predict mean values for the coarse cells with given degree of accuracy. The accuracy depends on the number of fine cells in the coarse resolution. Such an analysis must be followed by any application of the radiative transfer equation to a problem.

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